



KNOX
GRAMMAR
SCHOOL

STATE

DA VINCI DECATHLON 2022

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS
IN YEARS 5 & 6



MATHEMATICS

TEAM NUMBER _____

1	2	3	4	5	6	Total	Rank
/18	/8	/24	/18	/6	/4	/78	

Complete the above table with question numbers and marks as required.

QUESTION 1: PATTERN CONUNDRUMS (18 MARKS)

Throughout Mathematics there are a number of fundamental patterns whose results are used widely in many applications. Several patterns are shown below, using the part of the pattern demonstrated finish off the pattern and explain what it is.

- a) Pascals triangle is a key pattern used to determine the coefficients of a binomial expression when they are expanded. These expressions are fundamental to calculating probability combinations. Using the information shown, fill in the shaded boxes in the bottom two rows. (6 marks)

				1						
				1		1				
			1		2		1			
		1		3		3		1		
	1				6				1	
1										1

- b) The Fibonacci pattern is a sequence of numbers developed by Fibonacci in 1202. It was initially used for understanding population growth but it is now often used in computer algorithms particularly in search functions. Below is the first five numbers in the sequence, using this write the next three and explain the pattern. (4 marks)

0	1	1	2	3	5			
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- c) Another form of pattern is a Geometric pattern, this is often far simpler and can be applied to several scenarios. Below are two geometric patterns, complete the last 3 numbers for each (either as a fraction or decimal) and explain the pattern. (8 marks)

7	$\frac{63}{4}$	$\frac{567}{16}$	$\frac{5103}{64}$			
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1668	417	104.25	69.5			
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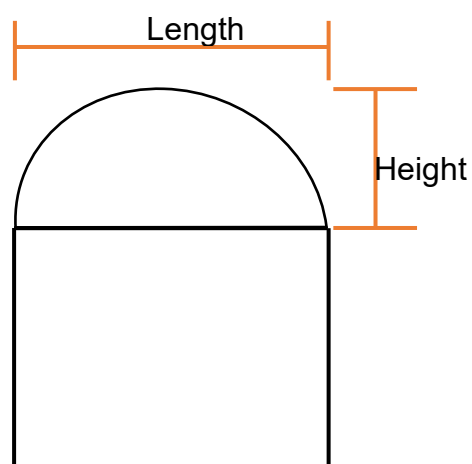
QUESTION 2: PATTERNS IN HISTORY

(8 MARKS)

Patterns are seen everywhere around us. In ancient architecture this is very prominent. Take the below photo of this aqueduct that the Romans built:



1. The arches for this aqueduct using a set of ratios. The length of the span: height of the span is a 2:1 ration as shown in the next diagram.



2. The first two rows of arches that compose the aqueduct are the same. The base arch height is 30m long.
3. There is another ratio where thickness of columns: length of the arch span is in a ratio of 1:5.
4. If we consider the arch length and one column length on the bottom row of the aqueduct, the top row has four arches and five columns for this same distance.
5. Otherwise, the third row of arches follow the same ratios as the bottom two rows.

What is the height of the span for one of the arches in the top row?

You may assume that all columns are exactly in line with each other.

8 MARKS

QUESTION 3: SALES PATTERNS

(24 MARKS)

You are a senior manager in charge of operations of Southern Springs Shopping Centres. You rent out spaces for food court and general retail. You charge \$350 per square metre per month for the food court area and \$695 per square metre for retail stores. If a merchant wishes to hire two or more store fronts, then they can receive a 7.5% discount on the normal total cost.

- a. Fried Chickedees hires a food court area and needs 15 square metres. What is the cost per year?

3 MARKS

- b. If one Southern Springs Shopping Centre has a total available rental space of two thousand square metres, what is the maximum income you can make per year?

4 MARKS

- c. For 2021, you decide a quarter of the 2,000 metres need to be devoted to a food court as this helps to attract customers. How much less money do you make per year as compared to having no food court?

4 MARKS

- d. 25% of the retail space is now devoted to the food court, but you find the food court area in 2021 was struggling and so you decide to offer a 10% discount to all food retailers in 2022. How much less money are you now making in 2022 compared to 2021?

4 MARKS

- e. You want to make up the loss in income from part (d) by increasing the rent to the retail stores. How much do you need to increase their rent per month?

3 MARKS

- f. A person contacts you for a quote where they wish to rent a 25 square meter retail store and a 15 metre food court store. What is your quote given you to follow the rules mentioned above?

6 MARKS

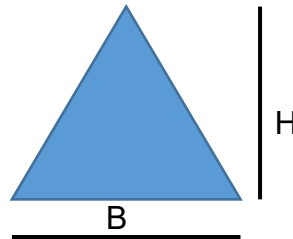
QUESTION 4: CITY BLOCK PATTERNS (18 MARKS)

Modern city blocks are often seen as square shapes placed throughout the city. However, there are a number of different possible designs that you can use such as triangular, circular, square, hexagonal and rhombuses.

Below are the sizes of the different potential city blocks.

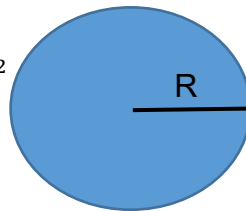
- Triangular (equilateral)

- $H = 52\text{m}$
- $B = 60\text{m}$



- Circular

- $R = 30\text{m}$
- $\text{Area} = 3.14 \times R^2$



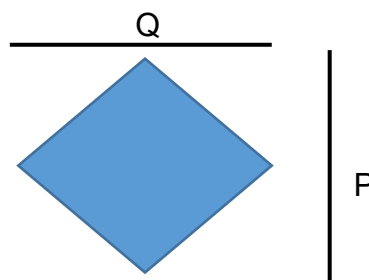
- Square

- $L = 60\text{m}$



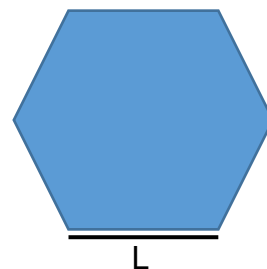
- Rhombus

- $P = 50\text{m}$
- $Q = 50\text{m}$



- Hexagonal

- $L = 40\text{m}$
- $\text{Area} = \frac{3\sqrt{3}}{2} \times L^2$



- a) Calculate the area of each of the shapes above and clearly note which area is related to which block shape.

10 MARKS

Below are two areas which the city is looking to place city blocks within. The city is going to use city blocks of the sizes calculated in part A. Assume that each city block fits the same number of houses, so the city block that fits the most number of times in each of the areas below is considered to be the 'most efficient'. Using the areas calculated in part A determine for each of the shapes in part b and c which city block is the most efficient. Assume that you can have fractional parts of the city blocks ie: can have $\frac{1}{5}$ of a circle block. In order to get full marks you must show your working.

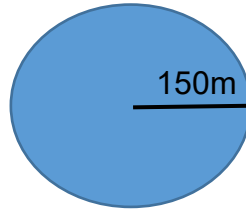
b) Which block shape is the most efficient in the square shape below?



4 MARKS

c) Which block shape is the most efficient in the circular shape?

4 MARKS



QUESTION 5: GAMES PROBABILITY (6 MARKS)

Billy, Ned and Abby played a round robin tennis tournament, where after each individual game the winner stays on whilst the person who lost swaps out with another person. At the end of several games Abby is exhausted after playing the last seven games whilst Billy who is better rested writes a list of how many games each person played.

- Billy 8 games
- Abby 12 games
- Ned 14 games

Hint: How many games were played in total?

- a) Who won the fourth game and who did they win against?

6 MARKS

QUESTION 6: CHAIN PATTERNS

(4 MARKS)

There are five patterned chains, which need to be combined into a larger single loop. In each patterned chain there are four individual links. Each link can be broken (and then resealed) to attach two other links together.

- a) What is the minimum number of the links that need to be broken to create a single chain of 20 links?

4 MARKS

END OF PAPER



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- a) Pascals triangle is a key pattern used to determine the coefficients of a binomial expression when they are expanded. These expressions are fundamental to calculating probability combinations. Using the information shown, fill in the shaded boxes in the bottom two rows; (6 marks)

				1						
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

6 marks	1 mark for each correct answer, if there is a carry through error than 0.5 marks can be given to the result.
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- b) The Fibonacci pattern is a sequence of numbers developed by Fibonacci in 1202. It was initially used for understanding population growth but it is now often used in computer algorithms particularly in search functions. Below is the first five numbers in the sequence, using this write the next three and explain the pattern; (4 marks)

0	1	1	2	3	5	8	13	21
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3 marks	1 mark for each correct answer.
1 mark	1 mark for explanation.

- c) Another form of pattern is a Geometric pattern, this is often far simpler and can be applied to several scenarios. Below are two geometric patterns, complete the last 3 numbers for each (either as a fraction or decimal) and explain the pattern. (8 marks)

7	$\frac{63}{4}$	$\frac{567}{16}$	$\frac{5103}{64}$	$\frac{45927}{256}$	$\frac{413343}{1024}$	$\frac{3720087}{4096}$
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1668	417	104.25	26.06	6.516	1.629	0.407
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6 marks	1 mark for each correct answer.
2 marks	1 mark for explanation.

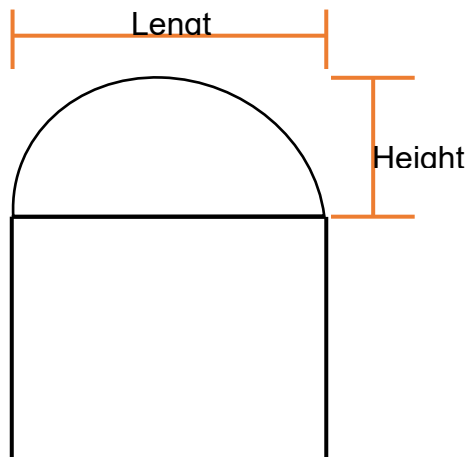
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5. Otherwise, the third row of arches follow the same ratios as the bottom two rows. What is the height of the span for one of the arches in the top row?

You may assume that all columns are exactly in line with each other. (8 marks)

Row 1 and 2: Base arch length = $30 \times 2 = \mathbf{60m}$

Length of span + the column = $60 + 12m = \mathbf{72 m}$

Row 3: Total length is 72m but for 4 columns (C) and 5 arch span lengths (L)

$72 = 4C + 5L$ where $c = 1/5$ of l so $72 = 5/5L + 4L = 5L$

Therefore, $72 = 5L$ or $L = \mathbf{14.4m}$

Height of each of the arches in top row (row 3) = $14.4 \text{ m} / 2 = \mathbf{7.2m}$

2 marks	For each correct line of working that gives the correct answer in BOLD .
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QUESTION 3: SALES PATTERNS

(24 MARKS)

You are the senior manager in charge of operations of Southern Springs Shopping Centres. You rent out spaces for food court and general retail. You charge \$350 per square metre per month for the food court area and \$695 per square metre for retail stores. If a merchant wishes to hire two or more store fronts, then they can receive a 7.5% discount on the normal total cost.

- (a) Fried Chickedees hires a food court area and needs 15 square metres. What is the cost per year?

Cost = \$350 x 15 meters square x 12 months = **\$63,000 per year.**

2 mark	Correct eventual answer (number).
2 marks	Correct working and / or units are provided.

- (b) If one Southern Springs Shopping Centre has a total available rental space of two thousand square metres, what is the maximum income you can make per year? Show your thinking in your calculations and reasoning.

Total potential income = 2,000 x \$695 (which is the highest rate per metre square) = \$1,390,000 per month.
Per year this is **\$16,680,000.**

2 marks	Correctly identifies 4 that all space should be retail space.
2 marks	Correctly calculates the amount of income.

- (c) For 2021, you decide a quarter of the 2,000 metres need to be devoted to a food court as this helps to attract customers, how much less money do you make per year as compared to having no food court? Show your thinking in your calculations and reasoning.

25% x 2,000m² = 500m² If the 500m² is devoted to food court = \$175,000
If the 500m² is devoted to retail = \$347,500
Therefore, if 25% is devoted to a food court the owner will earn \$172,500 less per month.
The question refers to per year and therefore we multiply this by 12 to get **\$2,070,000 less per year.**

2 marks	Correct calculations for comparing retail and food court. NB a few different methods are possible.
2 marks	Correctly gives the right answer PER YEAR.

- (d) 25% of the retail space is now devoted to the food court, but you find the food court area in 2021 was struggling and so you decide to offer a 10% discount to all food retailers in 2022. How much less money are you now making in 2022 compared to 2021? Show your thinking in your calculations and reasoning.

In 2021 you had $500\text{m}^2 \times \$350/\text{month}/\text{m}^2 \times 12 \text{ months} = 2,100,000$
 In 2022 you offer a 10% discount. 10% of 2,100,000 is **\$210,000**.

2 marks	Correct number.
2 marks	Calculations are provided that support the answer.

(e) You want to make up the loss in income from part (d) by increasing the rent to the retail stores. How much do you need to increase their rent per month? Show your thinking in your calculations and reasoning.

\$210,000 more is needed to cover the discount over 12 months.
That means \$210,000 (divided by 12) is \$17,500 needed each month.
The 17,500 extra per month will be spread across 1,500m².
That means the increase in cost per month for retail stores increases **\$11.67 / month / m².**
Students may express this as the cost to a retail store of **\$706.67 / month / m².**

2 marks	Correct number.
2 marks	Calculations are provided that support the answer.

(f) A person contacts you for a quote where they wish to rent a 25 square metre retail store and a 15-metre food court store. What is your quote given you must follow the rules mentioned above? Show your thinking in your calculations and reasoning.

$25\text{m}^2 \times \$695/\text{m}^2 \text{ per month} = \$17,375 \text{ per month.}$
 $15 \text{ m}^2 \times \$350/\text{m}^2 \text{ per month} = \$5,250 \text{ per month.}$
Discount = 7.5%. Therefore only paying 92.5% of the normal cost
 $= 0.925 \times (\$17,375 + \$5250) = \mathbf{\$20,928.13 \text{ cents}}$ (rounded up).

2 marks	Correct number.
2 marks	Calculations are provided that support the answer.

QUESTION 4: CITY BLOCK PATTERNS (18 MARKS)

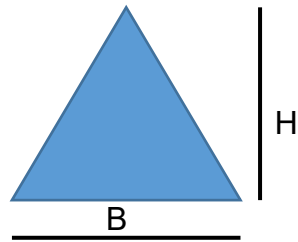
Modern city blocks are often seen as square shapes placed throughout the city. However, there are a number of different possible designs that you can use such as triangular, circular, square, hexagonal and rhombuses.

Below are the sizes of the different potential city blocks;

a) Triangular (equilateral)

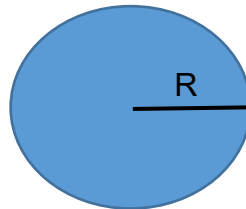
a. $H = 52\text{m}$

b. $B = 60\text{m}$



b) Circular

a. $R = 30\text{m}$



c) Square

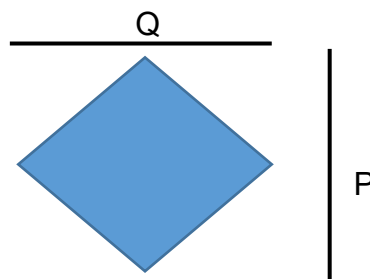
a. $L = 60\text{m}$



d) Rhombus

a. $P = 50\text{m}$

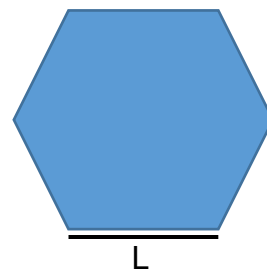
b. $Q = 50\text{m}$



e) Hexagonal

a. $L = 40\text{m}$

b. $\text{Area} = \frac{3\sqrt{3}}{2} \times L^2$



- a) Calculate the area of each of the shapes above and clearly note which area is related to which block shape. (10 marks)

$$\text{Triangle area} = \frac{1}{2} \times 52 \times 60 = 1560 \text{ m}^2$$

$$\text{Circular area} = 3.14 \times 30^2 = 2826 \text{ m}^2$$

$$\text{Square area} = 60 \times 60 = 3600$$

$$\text{Rhombus area} = 50 \times 50 \times 0.5 = 1250 \text{ m}^2$$

$$\text{Hexagon area} = \frac{3\sqrt{3}}{2} \times 40^2 = 4156.9 \text{ m}^2$$

2 marks	For each correct calculation of area. If incorrect formula was used, or correct formula and incorrect calculation 1 mark is deducted.
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- Which block shape is the most efficient in the square shape below? (4 marks)



Area = $300 \times 300 = 90000 \text{ m}^2$

Number of city blocks for each;

Triangle = $90000 / 1560 = 57$

Circular = $90000 / 2826 = 31$

Square = $90000 / 3600 = 25$

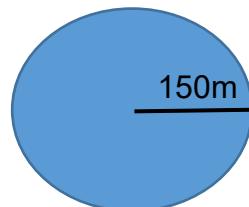
Rhombus = $90000 / 1250 = 72$

Hexagon = $90000 / 4156.9 = 21$

Hence the rhombus can fit the most.

1 mark	Correctly calculates the area of the space to put the blocks in
2 marks	Correctly uses area to determine how many blocks of each shape can be placed
1 mark	Correctly identifies rhombus as the most efficient shape

- Which block shape is the most efficient in the circular shape? (4 marks)



Area = $3.14 \times 150 \times 150 = 70650 \text{ m}^2$

Number of city blocks for each;

Triangle = $70650 / 1560 = 45.3$

Circular = $70650 / 2826 = 25$

Square = $70650 / 3600 = 19.6$

Rhombus = $70650 / 1250 = 56.52$

Hexagon = $70650 / 4156.9 = 16.996$

Hence the rhombus can fit the most.

1 mark	Correctly calculates the area of the space to put the blocks in
2 marks	Correctly uses area to determine how many blocks of each shape can be placed
1 mark	Correctly identifies rhombus as the most efficient shape

QUESTION 5: GAMES PROBABILITY (6 MARKS)

Billy, Ned and Abby played a round robin tennis tournament. Where after each individual game the winner stays on whilst the person who lost swaps out with another person. At the end of several games Abby is exhausted after playing the last seven games whilst Billy who is better rested writes a list of how many games each person played.

1. Billy 8 games
2. Abby 12 games
3. Ned 14 games

Hint: How many games were played in total?

1. How many games were played? (2 marks).
2. Provide a table that showed who played each of the games. (2 marks).
3. Who won the fourth game and who did they win against? (2 marks).

(6 marks in total)

When tallying up the total games played there were 17 games. As it is a round robin every players plays at least every other game. So each player has at least 8 games.

As Abby has played the last seven games she must have played games 11-17, then by deducing when the others could have played the following table is determined;

N	B	N	B	N	B	N	B	N	B	N	B	N	B	N	B	N
A	N	A	N	A	N	A	N	A	N	A	A	A	A	A	A	A

Thus Billy and Ned played the fourth game.

2 marks	Correctly identifies total number of games
2 marks	When solving identifies that the final games are all played by Abby
2 marks	Correctly identifies who played the fourth game

QUESTION 6: CHAIN PATTERNS

(4 MARKS)

There are five patterned chains which need to be combined into a larger single loop. In each patterned chain there are four individual links, each link can be broken then used to attach two other links together.

- What is the minimum number of the links that need to be broken to create a single chain of 20 links? (4 marks)

If only one link is broken on each chain, then 5 are needed to link the chains together.

However, the smallest number it can be done in is 4 links. This is achieved by breaking all of the links in one of the chains and then using these broken links to combine the remaining four links together.

4 marks	Correctly identifies 4 links
2 marks	If 5 is instead identified.

END OF PAPER