KNOX
GRAMMAR
SCHOOL

STATE

DA VINCI DECATHLON 2021

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS IN YEARS 5 & 6

MATHEMATICS

TEAM NUMBER _____

1	2	3	4	5	6	7	8	Total	Rank
/13	/10	/3	/5	/9	/3	/4	/8	/55	

QUESTION 1 – JUICE BOX (13 MARKS)

Joseph is selling a new fruit juice in a cubic container. He ships the new fruit juice containers in one large cubic box. Within that box there are 192 small cubes that touch exactly four other small cubes face-to-face.

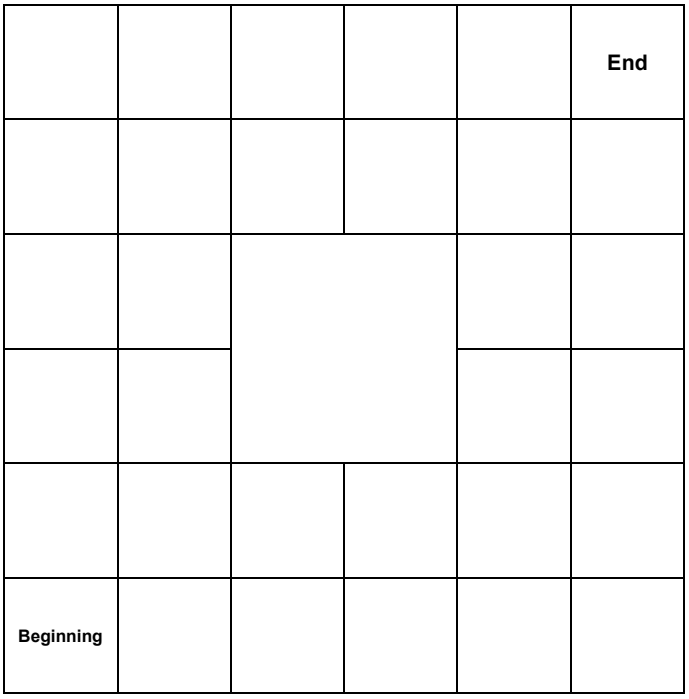
(a) By considering where in a larger cube the small cubes would only touch four other cubes, calculate home many small juice cubes in total can be stored in the large cubic box. (4 marks)

(b) Joseph is not satisfied with that quantity, so redesigns the large box to fit 6000 small juice cubes. He is told by the delivery company that each large box can be at most 1 tonne (t). Additionally, each cube's packaging is 25 g and the large box when empty weighs 1.5 kg. Knowing that 1 t = 1,016 L, 1 L = 1000 mL and 1 g = 1 mL, calculate the maximum possible capacity of each small juice cube to the nearest mL so that the delivery company will still accept the large box. Hint: start with the equation: total mass = total mass of all square cubes + mass of large box. (4 marks)

(c) After market research, Joseph has decided he prefers a volume of 220 mL per square cube. Assuming the same additional weights and restrictions as in (b), how many square cubes can Joseph fit into the large box now? (2 marks)

(d) Considering how these square cubes pack into the large box, is the approach Joseph took in (c) **sensible** and **accurate**? Explain your answer in words, supported with a calculation. (3 marks)

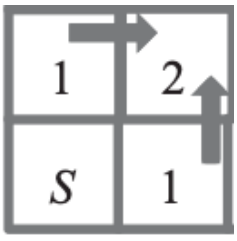
QUESTION 2 – CHAMPIONS OF CHANCE (10 MARKS)



Alex is designing a new board game for **3 players only**. There are 32 positions on the board. The bottom left position is labelled 'The Beginning' while the top right is labelled 'The End'. The aim of the game is to reach the end first (i.e. have the shortest route). To do so, each player takes turns in moving their token to a new position.

(a) A player is only allowed to land on each position once during a game (although many players can be on the same position at the same time). A player is permitted to move either vertically or horizontally only between positions. What is the greatest number of turns a player could make (i.e. the longest possible route) before winning? (1 mark)

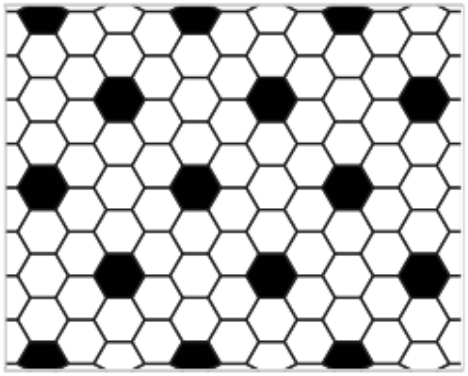
(b) It is known that the shortest path involves 10 turns (i.e. 9 other positions in addition to the beginning and end). How many possible ways can a player win using the shortest path? Hint: start by placing a number on each of the positions in the board that equals how many different ways there are to travel to that position along a route that starts at 'The Beginning'. The first three tiles around the beginning are illustrated below. Note though that these numbers won't be marked – they are only to aid you in reaching the answer! (3 marks)



(c) Alex wants to add a time estimate of the game length to the instructions. Using the longest and shortest route estimates from (a) and (b) and that on average it takes a person 40 seconds to take a turn, provide an estimate of the game length in the form (x – y minutes) where x is the shortest game length and y is the longest to the nearest 10 minutes (e.g. 10, 20, 30 etc) (4 marks).

(d) To launch the game, Alex is planning an 81-player knock-out tournament. Each match involves 3 players and only the winner of that round remains in the tournament, with the other two being knocked out. How many rounds are required until a winner will be determined? (2 marks)

QUESTION 3 – MOSQUITO MOSAIC (3 MARKS)



The above image shows a section of tiling on a new porch that is in total 3x3 metres large. A mosquito lands on one of the hexagonal tiles. What is the chance (purely based on the fraction of tiles that are black) that the fly lands on a black tile?

QUESTION 4 – CALCULATING CHANCE (5 MARKS)

Probability is the mathematics of chance – how likely an event is occur given all the events we know could happen. A simple formula is used at the core of probability:

$$\textbf{\textit{Probability of an event }}(\boldsymbol{P}) = \frac{\textbf{\textit{Total number of desired outcomes}}}{\textbf{\textit{Total number of possible outcomes}}}$$

(a) Jo's birthday is on 2 January. Explain using the above formula why the probability that Jo's birthday being on 2 January 2021 is 100%. (1 mark)

(b) What is the probability of picking a vowel from the word **EXPEDITIOUS**? (1 mark)

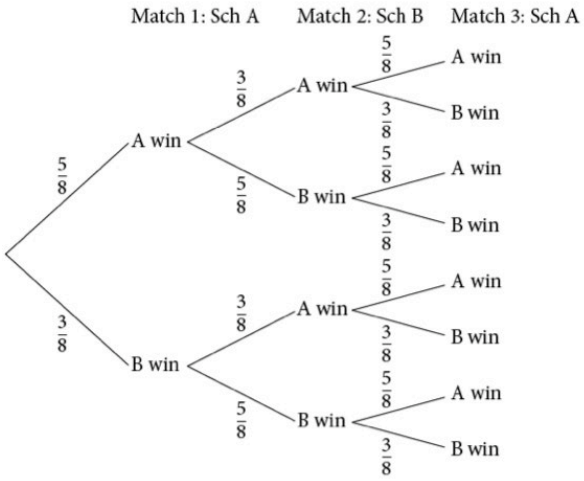
(c) What is the probability of rolling a multiple of 2 on a normal die? (1 mark)

(d) If a die has 20 sides what is the probability of rolling a 4 **or** a number below 4? (2 marks)

Hint: $Probability\ of\ rolling\ a\ number\ greater\ than\ 17\ =\ P(18) + P(19) + P(20)$

QUESTION 4 – DEPENDENT DILEMMAS (9 MARKS)

When probability events **depend** on each other, you multiply rather than add the chances of each event. For example, if the chances of selecting a red token from a bag is 30% and the chances of selecting a blue token are 20%, the chance of selecting a red then blue when the tokens are replaced in the bag after each draw is 30% x 20% = 6%.



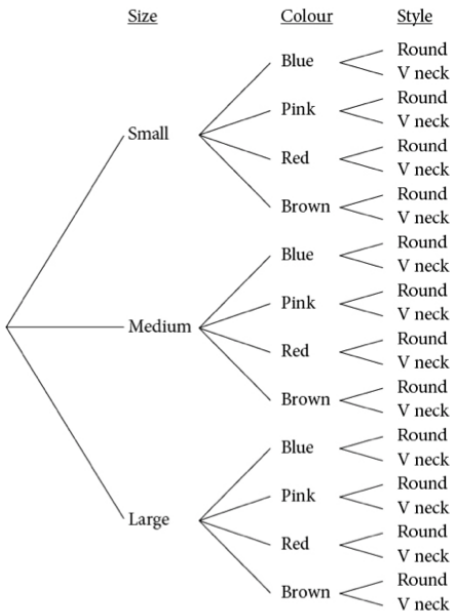
A tennis tournament is taking place and the odds of winning each match are indicated above for players A and B.

(a) Explain why the probability of A winning all three matches is 14.6%. (1 mark)

(b) Describe, using the formula in Question 4, why the probability of A not winning all three matches is 85.4%. (1 mark)

(c) List all the 'routes' along the image above that result in A winning two matches and B winning one match. (2 marks)

(d) What is the probability that A wins two matches and B wins one? Hint: you will need to also use the idea in Question 4 (d) to answer this! (3 marks)



(e) Cheryl is at a store buying new clothes. The above tree shows all the options she has for selecting a new shirt. The chances of selecting the options at each 'junction' are equal. What is the probability that Cheryl will end up with a pink shirt? (2 marks)

QUESTION 5 – COIN CONUNDRUM (3 MARKS)

Andrew placed some 20-cent coins on a checkout counter. 50% of the coins were 'tails' up. Andrew then turned over 6 of the coins, after which 1/3 of the coins were then 'tails' up. How many coins in total are on the counter?

QUESTION 6 – CHANCE MEET UP? (4 MARKS)

Four friends are hoping to meet for dinner, but they can't seem to schedule a convenient time for all. They know the following:

Alice is unable to meet on Tuesdays, Wednesdays or Saturdays

Bob is available on Mondays, Wednesdays and Thursdays

Charlie is busy on Mondays and Thursdays.

Doug is free Mondays, Tuesdays and Fridays.

No one is available on Sunday.

(a) What is the largest number of people that could meet and on which day? (1 mark)

(b) Can Charlie meet anyone on Saturday? (1 mark)

(c) Is there a way for Charlie to meet up with all the other three friends in his group at some point in the week? Explain. (2 marks)

QUESTION 7 – INSTRUMENT INVESTIGATION (8 MARKS)

While the above chances were easy to work with, this final question has a few extra variables. By completing the table below, work out which instrument each of the four students play and how old each student is. Write your answers in the appropriate spaces from (a) to (d). Hint: start by placing ticks/crosses in the boxes based on the immediate clues, then start to consider deeper logic to continue crossing and ticking.

(i) Andy is learning to play the Violin
(ii) The student learning about the Trumpet is 2 years younger than Don
(iii) Beth is either 6 or 8 years of age
(iv) The student who is 7 is learning the Flute

			Instruments				Age			
			Violin	Flute	Drums	Trumpet	6	7	8	9
Student	Andy									
	Beth									
	Clarise									
	Don									
Age	6									
	7									
	8									
	9									

(a) Andy
(b) Beth
(c) Clarise
(d) Don