KNOX GRAMMAR SCHOOL

STATE

DA VINCI DECATHLON 2021

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS IN YEARS 5 & 6

da Vinci Decathlon® EXCITE ENRICH CHALLENGE An initiative of Knox Grammar School

MATHEMATICS

TEAM NUMBER _____

1	2	3	4	5	6	7	8	Total	Rank
/13	/10	/3	/5	/9	/3	/4	/8	/55	

Note: these marks are guides. Digital marking may require full awarding if points with limited working out.

QUESTION 1 – JUICE BOX (13 MARKS)

Joseph is selling a new fruit juice in a cubic container. He ships the new fruit juice containers in one large cubic box. Within that box there are 192 small cubes that touch exactly four other small cubes face-to-face.

(a) By considering where in a larger cube the small cubes would only touch four other cubes, calculate home many small juice cubes in total can be stored in the large cubic box. (4 marks)

1 mark	The cubes not on an edge touch 5 faces, and internal cubes touch 6. The cubes that **touch four others lie along an edge and are not at a corner (which only touch 3) (only the bold is needed)**
1mark	Since cubes have 12 edges, 192 / 12 = 16. Therefore, each edge has 16 small cubes that touch 4 other cubes
1 mark	Considering the corner cubes also, each edge must have 18 cubes
1 mark	Therefore, total number of cubes is 18 x 18 x 18 = **5832**

(b) Joseph is not satisfied with that quantity, so redesigns the large box to fit 6000 small juice cubes. He is told by the delivery company that each large box can be at most 1 tonne (t). Additionally, each cube's packaging is 25 g and the large box when empty weighs 1.5 kg. Knowing that 1 t = 1,016 L, 1 L = 1000 mL and 1 g = 1 mL, calculate the maximum possible capacity of each small juice cube to the nearest mL so that the delivery company will still accept the large box. Hint: start with the equation: total mass = total mass of all square cubes + mass of large box. (4 marks)

1 mark	Conversion: 1,016 L = 1016000 mL = 1016000 g
1mark	Formula: 1016000 = 6000 x 25 + 6000 x (liquid capacity) + 1500 g

1 mark	Re arranging liquid capacity = 864500/6000
1 mark	Therefore, each juice container can take 144 mL (note 145 is incorrect, as the question asks to the nearest mL and if rounded to 145 this would exceed the 1 tonne limit).

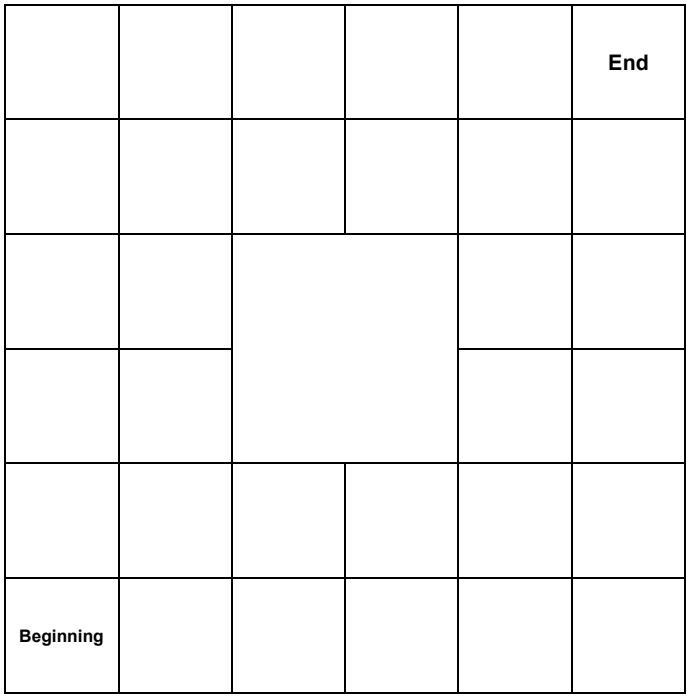
(c) After market research, Joseph has decided he prefers a volume of 220 mL per square cube. Assuming the same additional weights and restrictions as in (b), how many square cubes can Joseph fit into the large box now? (2 marks)

1 mark	1016000 = 6000 x 25 + amount of cubes x 220 + 1500 g
1 mark	Re arranging amount of cubes = 3292.545
1 mark	Therefore, Joseph could fit 3292 cubes (not 3293).

(d) Considering how these square cubes pack into the large box, is the approach Joseph took in (c) **sensible** and **accurate**? Explain your answer in words, supported with a calculation. (3 marks)

1 mark	The cube root of 3293 = 15.78 (i.e. not a square of cube number)
1 mark	Not sensible: This means that the squares would be unevenly packed, and space would remain at the top that needs additional packaging.
1 mark	Not accurate: This means less cubes could be packed than calculated in (c).

QUESTION 2 – CHAMPIONS OF CHANCE (10 MARKS)

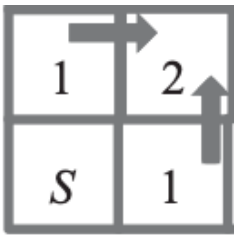


Alex is designing a new board game for **3 players only**. There are 32 positions on the board. The bottom left position is labelled 'The Beginning' while the top right is labelled 'The End'. The aim of the game is to reach the end first (i.e. have the shortest route). To do so, each player takes turns in moving their token to a new position.

(a) A player is only allowed to land on each position once during a game (although many players can be on the same position at the same time). A player is permitted to move either vertically or horizontally only between positions. What is the greatest number of turns a player could make (i.e. the longest possible route) before winning? (1 mark)

30 tiles so max of 31 turns, however being in both corners it is not possible to form a route that runs along every position, so the maximum number of turns is 30 (covering 29 of the tiles).

(b) It is known that the shortest path involves 10 turns (i.e. 9 other positions in addition to the beginning and end). How many possible ways can a player win using the shortest path? Hint: start by placing a number on each of the positions in the board that equals how many different ways there are to travel to that position along a route that starts at 'The Beginning'. The first three tiles around the beginning are illustrated below. Note though that these numbers won't be marked – they are only to aid you in reaching the answer! (3 marks)



From the Beginning there are two starting moves (up or right). Along the edges of the game there is only one way to travel. For each position not along the edge, it can only be reached in two ways.

1	6	11	16	26	52
1	5	5	5	10	26
1	4			5	16
1	3			5	11
1	2	3	4	5	6
S	1	1	1	1	1

The End is 5 steps up and 5 steps to the right of the Beginning. Each of the moves can only be up or to the right so there must be **52 paths from the Beginning to the Finish (3 marks for correct answer)**

(c) Alex wants to add a time estimate of the game length to the instructions. Using the longest and shortest route estimates from (a) and (b) and that on average it takes a person 40 seconds to take a turn, provide an estimate of the game length in the form (x – y minutes) where x is the shortest game length and y is the longest to the nearest 10 minutes (e.g. 10, 20, 30 etc) (4 marks).

Shortest = 10 turns. Therefore, there would be 9x3 + 1 turns taken so minimum time = 28x40 seconds = 18.666667 minutes, **rounded to 20 minutes**

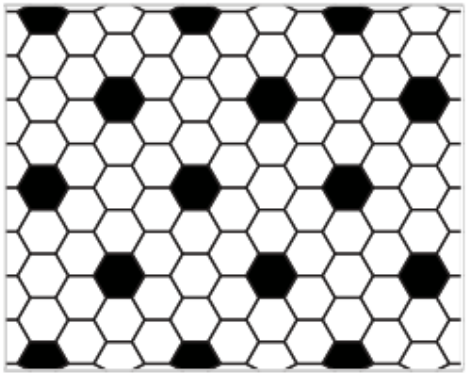
Longest = 30 turns. Therefore, 29x3 + 1 turns for the longest time to win, so maximum time = 58.6667 minutes, **rounded to 60 minutes**

Therefore, **20-60 minutes (2 marks for each answer, 1 mark for one answer if not rounded correctly, but don't double penalise a second rounding error (i.e. award 2/2).**

(d) To launch the game, Alex is planning an 81-player knock-out tournament. Each match involves 3 players and only the winner of that round remains in the tournament, with the other two being knocked out. How many rounds are required until a winner will be determined? (2 marks)

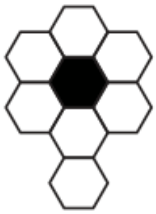
40 - Two players are knocked out as a result of one match. To leave a winner, 80 players must be knocked out. Therefore there must be 40 matches.

QUESTION 3 – MOSQUITO MOSAIC (3 MARKS)



The above image shows a section of tiling on a new porch that is in total 3x3 metres large. A mosquito lands on one of the hexagonal tiles. What is the chance (purely based on the fraction of tiles that are black) that the fly lands on a black tile?

Find the repeating pattern of one black to 7 white tiles, therefore 1/8 is the chance of landing on a black tile.



QUESTION 4 – CALCULATING CHANCE (5 MARKS)

Probability is the mathematics of chance – how likely an event is occur given all the events we know could happen. A simple formula is used at the core of probability:

$$\boldsymbol{Probability\ of\ an\ event\ (P) = \frac{Total\ number\ of\ desired\ outcomes}{Total\ number\ of\ possible\ outcomes}}$$

(a) Jo's birthday is on 2 January. Explain using the above formula why the probability that Jo's birthday being on 2 January 2021 is 100%. (1 mark)

There is one desired outcome and only one total possible outcome, so 1/1 = 100%

(b) What is the probability of picking a vowel from the word **EXPEDITIOUS**? (1 mark)

$$P(vowel) = \frac{6}{11} = 0.545 = 54.5\%$$

(c) What is the probability of rolling a multiple of 2 on a normal die? (1 mark)

$$P(2,4,6) = \frac{3}{6} = 0.5 = 50\%$$

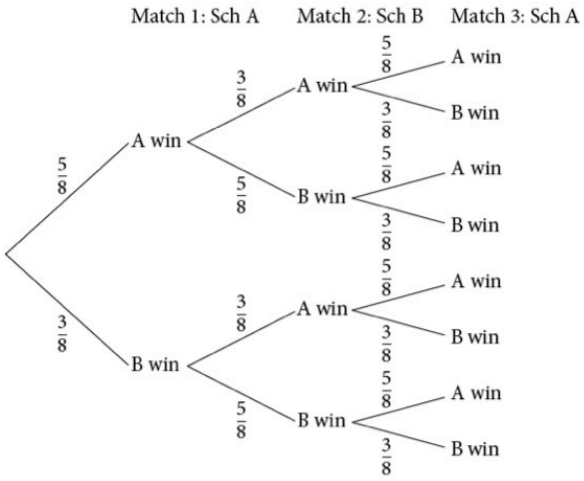
(d) If a die has 20 sides what is the probability of rolling a 4 **or** a number below 4? (2 marks)

Hint: $Probability\ of\ rolling\ a\ number\ greater\ than\ 17\ =\ P(18) + P(19) + P(20)$

$$P(4,3,2,1) = P(1) + P(2) + P(3) + P(4) = 4 \times \frac{1}{20} = \frac{1}{5} = 0.2 = 20\%$$

QUESTION 4 – DEPENDENT DILLEMAS (9 MARKS)

When probability events **depend** on each other, you multiply rather than add the chances of each event. For example, if the chances of selecting a red token from a bag is 30% and the chances of selecting a blue token are 20%, the chance of selecting a red then blue when the tokens are replaced in the bag after each draw is 30% x 20% = 6%.



A tennis tournament is taking place and the odds of winning each match are indicated above for players A and B.

(a) Explain why the probability of A winning all three matches is 14.6%. (1 mark)

5/8 x 3/8 x 5/8 = 14/6% (the events depend on each other)

(b) Describe, using the formula in Question 4, why the probability of A not winning all three matches is 85.4%. (1 mark)

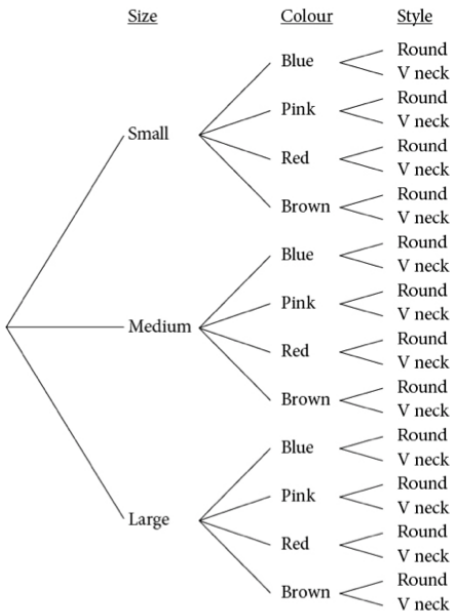
There are two outcomes, winning all 3 or not. If willing has a 14.6% chance, then the other option must be 1-14.6% = 85.4%

(c) List all the 'routes' along the image above that result in A winning two matches and B winning one match. (2 marks)

A-B-A
B-A-A (1 mark for each)

(d) What is the probability that A wins two matches and B wins one? Hint: you will need to also use the idea in Question 4 (d) to answer this! (3 marks)

5/8 x 5/8 x 5/8 + 3/8 x 3/8 x 5/8 = 33.2% (1 mark for equation of single route, 1 mark for adding and 1 mark for final answer)



(e) Cheryl is at a store buying new clothes. The above tree shows all the options she has for selecting a new shirt. The chances of selecting the options at each 'junction' are equal. What is the probability that Cheryl will end up with a pink shirt? (2 marks)

Options:
Small – pink
Medium – Pink
Large – Pink

Chances:
(1/3 x ¼)*3 = 25% (1 mark for reasoning and 1 mark for answer)

Students can also recognise that ¼ options in the tree involve pink, so by observation the chance is 25%.

QUESTION 5 – COIN CONUNDRUM (3 MARKS)

Andrew placed some 20-cent coins on a checkout counter. 50% of the coins were 'tails' up. Andrew then turned over 6 of the coins, after which 1/3 of the coins were then 'tails' up. How many coins in total are on the counter?

T = total coinage
T/2 = number of tails
(T/2)-6=T/3
Therefore,
T – 12 = 2T/3
3T – 36 = 2T

Therefore, there are 36 coins (2 marks, 1 mark for reasoning/logic process)

QUESTION 6 – CHANCE MEET UP? (4 MARKS)

Four friends are hoping to meet for dinner, but they can't seem to schedule a convenient time for all. They know the following:

Alice is unable to meet on Tuesdays, Wednesdays or Saturdays

Bob is available on Mondays, Wednesdays and Thursdays

Charlie is busy on Mondays and Thursdays.

Doug is free Mondays, Tuesdays and Fridays.

No one is available on Sunday.

(a) What is the largest number of people that could meet and on which day? (1 mark)

Monday!

(b) Can Charlie meet anyone on Saturday? (1 mark)

NO

(c) Is there a way for Charlie to meet up with all the other three friends in his group at some point in the week? Explain. (2 marks)

Yes (1 mark) – D Tuesday or Friday, B Wednesday and A Friday (1 mark – all needed)

QUESTION 7 – INSTRUMENT INVESTIGATION (8 MARKS)

While the above chances were easy to work with, this final question has a few extra variables. By completing the table below, work out which instrument each of the four students play and how old each student is. Write your answers in the appropriate spaces from (a) to (d). Hint: start by placing ticks/crosses in the boxes based on the immediate clues, then start to consider deeper logic to continue crossing and ticking.

(i) Andy is learning to play the Violin
(ii) The student learning about the Trumpet is 2 years younger than Don
(iii) Beth is either 6 or 8 years of age
(iv) The student who is 7 is learning the Flute

			Instruments				Age			
			Violin	Flute	Drums	Trumpet	6	7	8	9
Student	Andy									
	Beth									
	Clarise									
	Don									
Age	6									
	7									
	8									
	9									

(a) Andy – 9 years and violin
(b) Beth – 6 years old and Trumpet
(c) Clarise – 7 years old and Flute
(d) Don – 8 years old and Drums

1 mark for each correct age and instrument