KNOX
GRAMMAR
SCHOOL

STATE

DA VINCI DECATHLON 2021

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS IN YEARS 9, 10 & 11

MATHEMATICS

TEAM NUMBER _____

1	2	3	4	Total	Rank
/9	/16	/16	/9	/50	

QUESTION ONE

ROLL UP, ROLL UP

9 MARKS

When one thinks of **numbers** and **'chance'**, a dice is the first thing that comes to mind. Almost everyone, it can be said with certainty, will picture a **six-sided** one. Dice with more or less sides **do exist** but are **very uncommon,** and at the most they might have a few extra sides – perhaps 12 at the maximum.

Dice Lab, however, a small Arizonan company, had other ideas, and set about creating a 120-sided dice back in 2016. The mathematical name for this shape is a **disdyakis triacontahedron**, and this is the focus of the present task!

1. How many edges does a disdyakis triacontahedron have? _____**(1 mark)**

2. How many vertices does the shape have? _____ **(1 mark)**

3. What type of triangles are the faces? _____ **(1 mark)**

4. Which feature of the shape does the equation below represent? _____**(1 mark)**

$$\frac{180}{11}(5+4\sqrt{5})$$

5. Which feature of the shape does the equation below represent? _____**(1 mark)**

$$\frac{180}{11}\sqrt{179-24\sqrt{5}}$$

6. Complete the equation, which represents the number of possible combinations of numbers from 1 to 120 that could be placed on the shape, by inserting a number in the brackets **(3 marks)**.

$$N = 10^{(\quad)}$$

7. The above number is greater than the estimated number of atoms in the universe. True or false? _____ **(1 mark)**

QUESTION TWO

IN WITH A CHANCE

16 MARKS

Problems involving **probability** are among the most challenging in the mathematical world but also the most **practical.** Can you answer those below?

QUESTION ONE (3 MARKS)

One hundred people line up to board an airplane. Each has a boarding pass with assigned seat. However, the first person to board has lost his boarding pass and takes a random seat. After that, each person takes the assigned seat if it is unoccupied, and one of unoccupied seats at random otherwise. What is the probability that the last person to board gets to sit in his assigned seat?

QUESTION TWO (5 MARKS)

Mr. Smith works on the 13th floor of a 15 floor building. The only elevator moves continuously through floors 1, 2, . . . , 15, 14, . . . , 2, 1, 2, . . . , except that it stops on a floor on which the button has been pressed.

Assume that time spent loading and unloading passengers is very small compared to the travelling time. Mr. Smith complains that at 5pm, when he wants to go home, the elevator almost always goes up when it stops on his floor. What is the explanation?

Now assume that the building has n elevators, which move independently. Compute the proportion of time the first elevator on Mr. Smith's floor moves up.

QUESTION THREE (4 MARKS)

There are 64 teams who play single elimination tournament, hence 6 rounds, and you have to predict all the winners in all 63 games. Your score is then computed as follows: 32 points for correctly predicting the final winner, 16 points for each correct finalist, and so on, down to 1 point for every correctly predicted winner for the first round. (The maximum number of points you can get is thus 192.) Knowing nothing about any team, you flip fair coins to decide every one of your 63 bets. Compute the expected number of points.

QUESTION FOUR (4 MARKS)

You are a broker; your job is to accommodate your client's wishes without placing any of your personal capital at risk. Your client wishes to place an even $1,000 bet on the outcome of the World Series, which is a baseball contest decided in favour of whichever of two teams first wins 4 games. That is, the client deposits his $1,000 with you in advance of the series. At the end of the series he must receive from you either $2,000 if his team wins, or nothing if his team loses. No market exists for bets on the entire world 2 series.

However, you can place even bets, in any amount, on each game individually. What is your strategy for placing bets on the individual games in order to achieve the cumulative result demanded by your client?

QUESTION THREE

LUCK OF THE DRAW

16 MARKS

Everyone dreams of winning the **lottery**, however, the odds are stacked against us. This question will reveal just how **unlikely** a lottery win actually is.

QUESTION ONE (4 MARKS)

A standard lottery sees 6 numbers drawn in a row, from a pool of balls numbered 1 to 49, without replacement. Calculate the chance of **winning** this lottery (i.e. of picking the 6 numbers in any order).

QUESTION TWO (4 MARKS)

Now, calculate the chance of picking just **one** correct number!

QUESTION THREE (2 MARKS)

How about the chance of coming *oh so close* and picking **five** correct numbers?

QUESTION FOUR (SIX MARKS)

Below are details for six types of lotteries across the world – order them from **most to least likely to be won.**

- **Irish Lotto** – 6 numbers, 47 balls
- **PowerBall** – 5 numbers plus 1 bonus number, 69 balls plus 26 bonus balls
- **MegaMillions** – 5 numbers plus 1 bonus number, 70 balls plus 25 bonus balls
- **EuroJackpot** – 5 numbers plus 2 bonus numbers, 50 balls plus 10 bonus balls
- **Oz Lotto** – 7 numbers, 45 balls
- **EuroMillions** - 5 numbers plus 2 bonus numbers, 50 balls plus 12 bonus balls

Answers:

1. _____ (most likely to be won)
2. _____
3. _____
4. _____
5. _____
6. _____ (least likely to be won)

QUESTION FOUR

SIGNING OFF

9 MARKS

Probability involves a significant number of **mathematical signs and symbols**, many of which are rarely seen in any other fields. Are you able to **draw** those named in the table below (some, but not all of which, are probability-related)?

SIGN OR SYMBOL NAME	ANSWER (DRAWING)
Summation	
'is proportional to'	
Real numbers	
'because'	
Population mean	
Standard deviation	
Combination	
Permutation	
Delta	

END OF PAPER