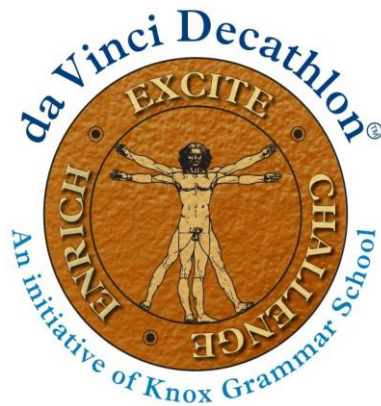


STATE

DA VINCI DECATHLON 2019

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS
IN YEARS 5 & 6



MATHEMATICS - SOLUTIONS

TEAM NUMBER _____

1	2	3	4	5	6	7	8	9	10	Total	Rank
/12	/6	/6	/11	/5	/4	/8	/3	/5	/12	/72	

QUESTION 1: RAPID RIVER

12 MARKS



- (a) A lake has three inlets of water flowing into it. One releases 1 litre of water into the lake ever 4 minutes. The second takes 8 minutes to release 1 litre. The third inlet release 500 mL of water every 30 minutes. On Monday at 9am, the lake contains 10,000 litres of water. When will the lake contain 10,100 litres of water (assuming the rate of flow into the river for each inlet is constant)? Hint: First consider how much water flow per hour. (5 marks)

MARKS	CRITERIA
3	Consider the amount of water per hour: 15L from inlet 1 7.5L from inlet 2 1L from inlet 3 Total = 23.5 litres
1	100L = 4.3 hours = 4 hours and 15 minutes (15.3 minutes exactly)
1	1:15pm.

- (b) In stormy weather, the inlets increase their water flow by 35%. Unfortunately, in a recent storm, the inlet that releases 1 litre of water every 4 minutes was blocked by a tree and no longer provides water to the lake. Assuming the lake is at 10,000 litres again, will it take more or less time for the lake to fill to 10,100 litres than in (a)? Provide detailed calculations in your answer. (3 marks)

MARKS	CRITERIA
1	Per hour only 8.5L now (before 3% increase). This is 35% more= 11.475L.
1	$100L/11.475 = 8.714$ hours
1	It will take, around 4.4 hours more (about double the time). Answer must include a number of 4.4 (+/-1) and also the percentage of approximately 200% (double the time). ½ a mark for each.

- (c) Jeff and Julia decide to go fishing on the lake. They row 1km from the shore before they hit a rock and create a hole in the bottom of the boat. The boat is leaking in water at a constant rate of 10 litres per minute. The boat will sink once it takes in 30 litres of water. Julia starts rowing back to the shore at a rate of 4 km/h while Jeff begins to take water out of the boat using a 100 mL bucket. What is the **slowest** rate that Jeff can take water out of the boat so they are to reach the shore without sinking? (3 marks)

MARKS	CRITERIA
1	It will take 15 minutes to reach the shore
1	$30/15 = 2$ L can enter the boat per minute
1	Since 10 L enter per minute, Jerry must remove $10-2 = 8$ litres per minute (80 buckets!).

- (d) After patching the boat Jeff and Julia return to the lake the following day. Their boat has a rope ladder hanging over its side in such a way that the lowest rung is 20cm from the surface of the lake. The rungs are 23cm apart. Due to recent storming, the inlets are raging with water and the lake rises 1m while they are fishing. How many rungs of the ladder will be underwater after the 1m rise? (1 mark)

1 mark – NONE – the boat will rise with the water!

QUESTION 2: SPOTTING SPIDERS

6 MARKS

Josh and Joanna are hiking and spot quite a few spiders while walking. They have entered a spider spotting competition, with the aim of spotting the greatest number of spiders. The following point system is used:

1. Each spider that is spotted is assigned a number, starting with 1.
2. The team then receives 10 points for every digit contained in the number. For example, if they spot two spiders then Josh and Joanna receive $10 + 10 = 20$ points. If 11 spiders were spotted, the 11th spider would award Josh and Joanna 20 points as it contains two digits.

Josh and Joanna managed to score 69,290. How many spiders did they spot?

MARKS	CRITERIA
1	9 spiders = 90 points
1	90 spiders with 10-99 assigned = 90 numbers = 1800 points
1	900 spiders (100-999) with three digits = 27,000 points
1	$69,290 - 28,890 = 40,400$ so the remaining must be from 4 digit numbers! There are 40 points assigned to these, so we need 1,010 numbers.
1	$1,010 + 999 = 2009$. Therefore, 2009 spiders were spotted.

QUESTION 3: THE BEST BARGAIN?

6 MARKS

- (a) A new green rose is on sale at Landscapes-R-us for 20% of its original price (\$30) and a further 10% off the reduced price is offered if you buy two. A second purple rose is on sale for 30% of its original price (\$30). Which is the better deal if you want to purchase two roses? Explain. (3 marks)

MARKS	CRITERIA
1	Value for green rose (per rose) = \$21.60
1	Value for purple rose (per rose) = \$21
1	Conclusion = purple rose is better deal.

- (b) Landscapes-R-Us has another sale on the next week for its sunflowers. The Germanic sunflower is 15% off followed by a \$90 rebate (return of money). The Nordic sunflower is the same price but is 25% off with no rebate. Bob saves \$15 when purchasing the Germanic sunflower instead of the Nordic sunflower. What is the original price of the sunflowers? (3 marks)

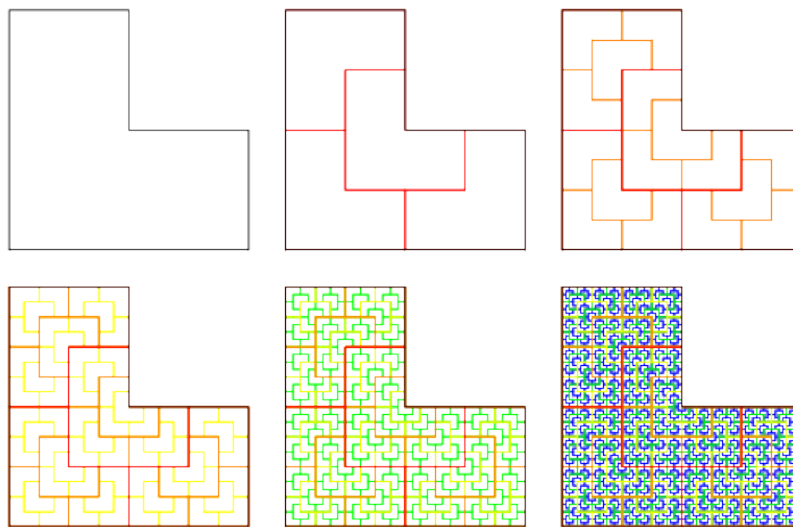
MARKS	CRITERIA
1	Price of Germanic sunflower = $0.85x - 90$ Price of Nordic sunflower = $0.75x$
1	We know that $0.85x - 90 + 15 = 0.75x$.
1	Therefore, $x = 750$ so the original price was \$750.

If algebra is not used, students may also reason that \$90 off is \$15 better than the additional 10% off. Therefore, additional 10% = \$75, so the original price is \$750.

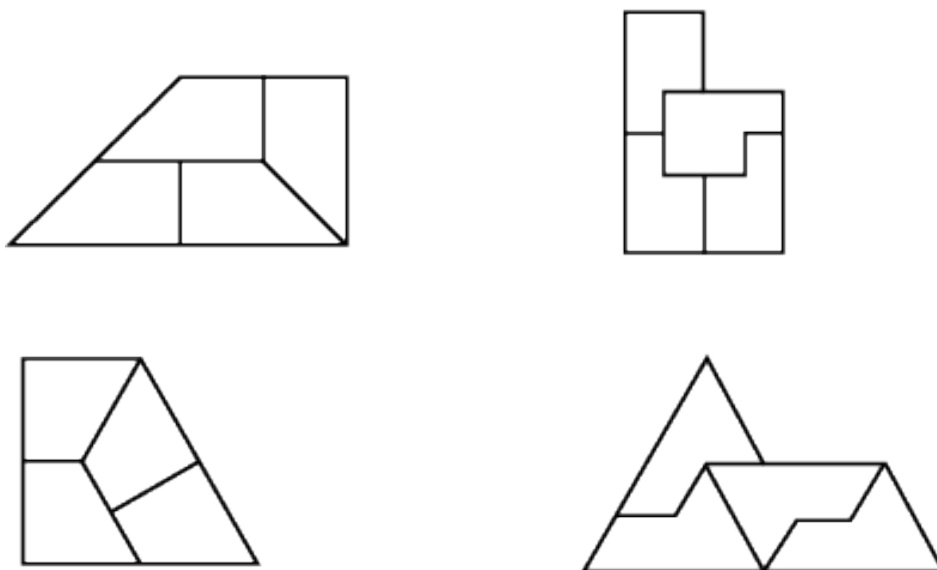
QUESTION 4: UNRAVELLING REP-TILES

11 MARKS

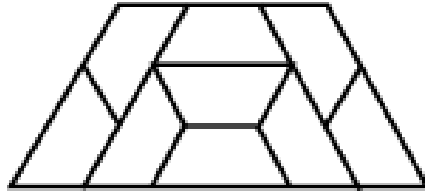
Reptiles appear throughout the native Australian landscape. Reptiles are self-replicating, and also often shed their entire outer layer of skin to reveal a new identical layer of skin below. Certain shapes have been named after reptiles, known as Rep-tiles, because they can be divided into copies of the original shape. Below is an example, illustrating that if you repeat the copying many times you can produce intricately patterned landscapes.



- (a) Are the following shapes rep-tiles? Demonstrate why or why not for each shape. (6 marks) **Yes for all – see division below. ½ mark for concluding yes to each (total of 2 marks). 1 mark for correct demonstration of rep-tile ability (see below).**

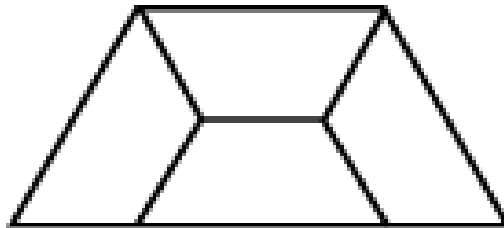


(b) Below is a rep-tile. When the shape is first repeated there are 9 smaller shapes inside the original. Illustrate how these shapes fit into the original rep-tile. (2 marks)



MARKS	CRITERIA
2	All correct
1	Attempt at creating a rep-tile but some errors

(c) Below is a rep-tile already with one repeat complete. Sketch the second repeat of the shape (see for example the top right shape on page 6). You may choose to redraw the shape at a larger scale if preferable to your team. (3 marks)



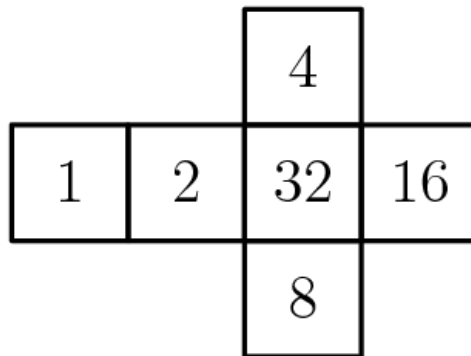
MARKS	CRITERIA
3	Detailed and accurate recreation of the above shape in each of the 4 smaller trapeziums
2	Correct but untidy
1	2-3 small errors or partly complete but accurate for the components that were completed

QUESTION 5:

CONSTRUCTING CUBES

5 MARKS

Three cubes are each formed from the pattern shown. A tower is made of the three cubes, one on top of each other, so that the 13 visible numbers have the **greatest possible sum**. Calculate what that sum is.



MARKS	CRITERIA
1	There will be 5 unseen faces. To maximise sum of numbers we want to minimise the sum of the 5 unseen faces.
1	When the cube is folded, 1/32; 2/16 and 4/8 are opposite pairs. 4+8 have the smallest sum. So the minimum = $12 + 12 + 1 = 25$
1	The top cube only has one face not showing. This must be a 1
1	As the maximum sum as $3 \times (32+16+8+4+2+1) = 189$
1	The maximum possible sum of the remaining 13 visible numbers = $189-25 = 164$.

QUESTION 6: LIGHTWORK

4 MARKS

A light is placed in the corner of a town square. It can illuminate the shaded region of the town square as shown below (diagram not to scale). The two lines from each corner of the square go to the middle of each of the opposite sides. What fraction of the area of the square is the shaded area?



Each of the unshaded triangles occupies a quarter of the square (as shown on the right) (1 mark for wording, 1 mark for visual illustration)

So the fraction of the room in which the camera can detect = $1 - (2 \times 1/4)$ (1 mark)

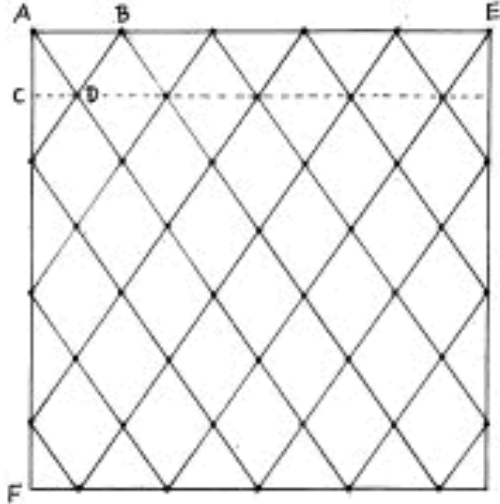
= $1/2$ (1 mark)

QUESTION 7: VINEYARD VARIANCE

8 MARKS

A square vineyard has a side length of 52m. Below is a representation of that vineyard using a scale of 1:500 (side length=10.4cm).

- (a) What are the maximum number of grape vine seedlings that can be planted if the seeds have to be at least nine metres apart? Illustrate the arrangement you have used to do this on the square below. Note: As they are seeds, they don't currently take up any space. You may also place seeds on the boundaries of the vineyard. Hint: The best arrangement has an isosceles triangle (two sides of equal length) between each three adjacent seeds (5 marks)

MARKS	CRITERIA
2	2 mark = 44 is the maximum number of seeds
1	1 mark = Arrangement so that there are 6 in one row, followed by 5, then 6 etc...
1	1 mark = 8 of these alternating rows
1	1 mark = tidy and accurate diagram 

(b) Estimate the distance between the seeds horizontally (1 mark)

10.4 m (this should be read from the scale diagram, not computed. Allow error of +/- 0.1 cm).

(c) Estimate the side lengths of the isosceles triangles between each three closest seeds (2 marks)

10.4m, 9.1 m (these should be read from the scale diagram, not computed. Allow error of +/- 0.1cm).

QUESTION 8: A SIMPLE TASK

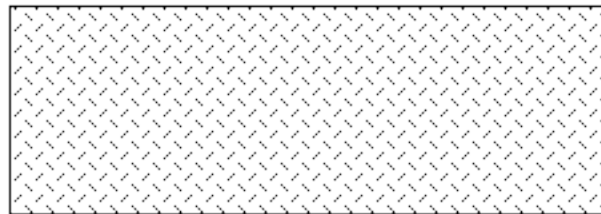
3 MARKS

Here is a light reprieve before the final two questions. Mark on the second shape how it can be cut into only two pieces so that it will fill the hole (top shape) exactly.

A hole:
(2m x 12m)



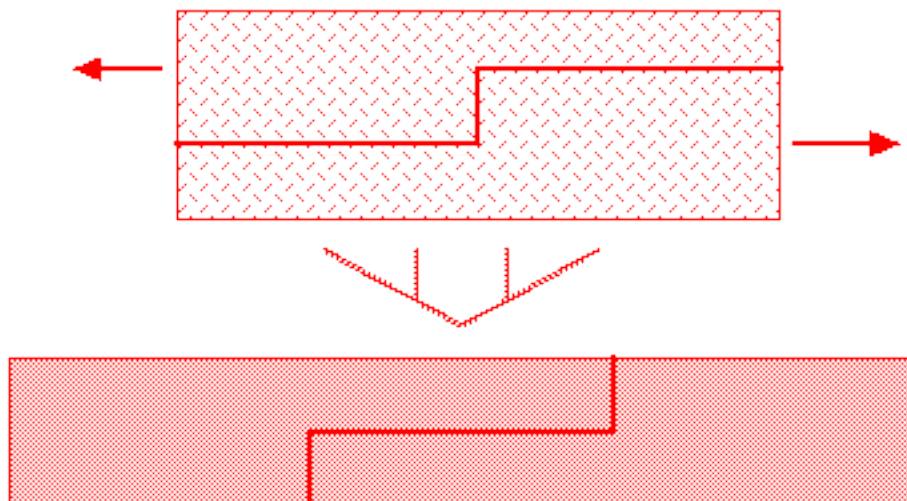
A board:
(3m x 8m)



3 marks = correct cut made

2 marks = attempt to form correct shape but untidy work

1 mark = only attempted to cut the shape in half.



QUESTION 9: ABC...123

5 MARKS

If $A = 1$, what is the value of B , C , D , E and F (all different digits) where:

$$ABC \times DEF = 123456$$

1 mark for each,

$$B = 9$$

$$C = 2$$

$$D = 6$$

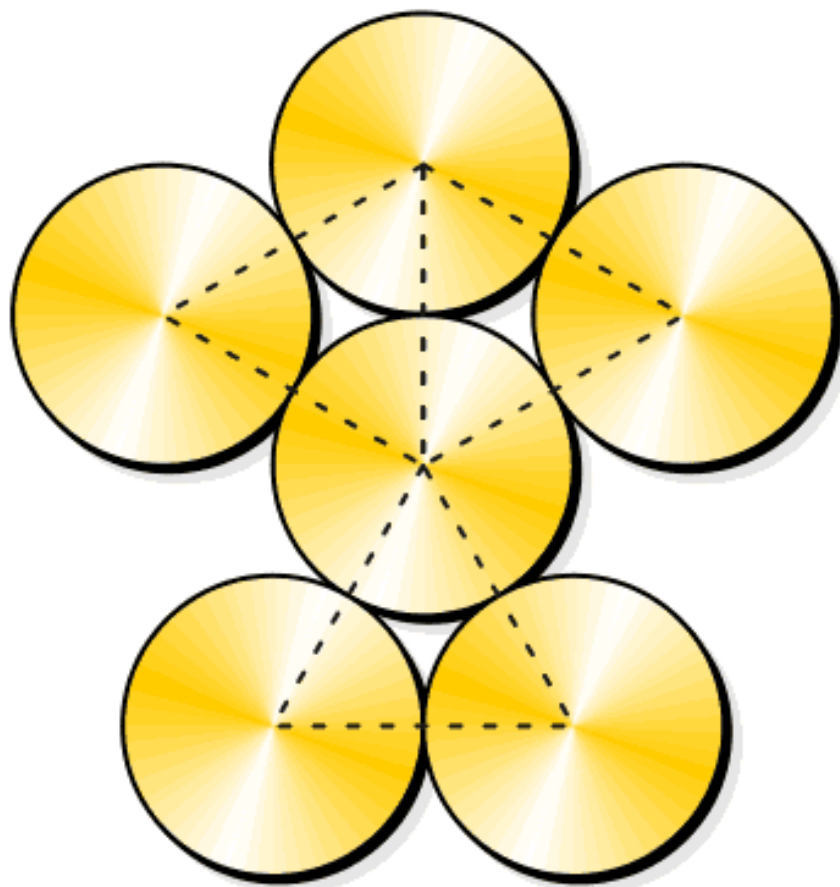
$$E = 4$$

$$F = 3$$

QUESTION 10: SOLAR CELL PREDICAMENT

12 MARKS

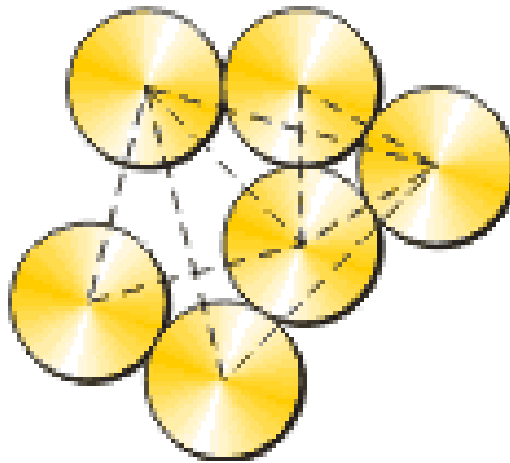
A set of six solar cells are arranged as shown below so that between them there are three identical equilateral triangles (all sides are of equal length).



This pattern has been favoured by the equilateralists in the town, but other more creative types want a change. You have been tasked by the local architect to move the cells into a new arrangement that still has three equilateral triangles, but this time of different sizes, that are formed by drawing lines between the centres of each circle.

The architect has given you strict rules. You may only move one cell at a time, and once moved it must touch at least to other cells.

- (a) Illustrate what the new arrangement will look like by sketching 6 new circles (at any scale you choose). Be careful, as it cannot just be a rotation or reflection of the current arrangement! Hint: the bottom left disc and middle discs don't change positions in the new arrangement (6 marks)



1 mark for the correct placement of each disc.

- (b) What are the minimum number of moves you need to make to reach this new arrangement? Use arrows on the original cell layout on page 14 to describe each move. (6 marks)

1 mark for five, then one mark for each correct move.

½ marks will be awarded for more than 5 movies, but less than 8 moves, arrangement.

