KNOX GRAMMAR SCHOOL

STATE

DA VINCI DECATHLON 2019

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS IN YEARS 7 & 8

MATHEMATICS

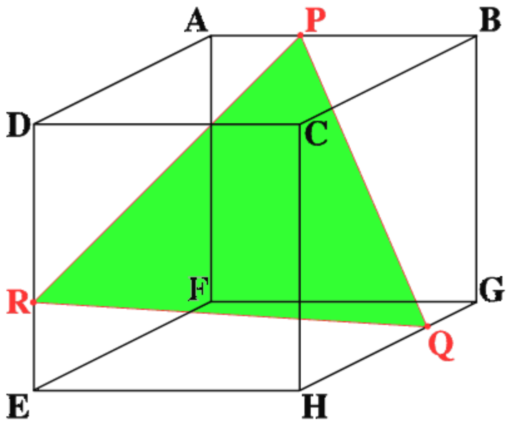
TEAM NUMBER ____________

1	2	3	4	5	6	7	8	Total	Rank
/9	/9	/6	/10	/15	/4	/10	/25	/88	

QUESTION ONE

TRIANGULAR TARPAULIN (9 MARKS)

Joseph, a landscape architect, wants to install a new tarpaulin in a cubic room. He creates a design, shown below, where point P is 1/3 along AB, point Q is 1/3 along GH and point R is 1/3 along ED. It is known for a right-angled triangle of sides *a*, *b* and *c* (the hypotenuse) that $a^2+b^2=c^2$.



(a) If the room is a cube of side length 3cm, calculate the length of BQ, PQ, QR and RP. (4 marks)

(b) What type of triangle is RPQ? (1 mark)

(c) Calculate the area of tarpaulin required to create the triangular tarpaulin PRQ designed by Joseph? (4 marks)

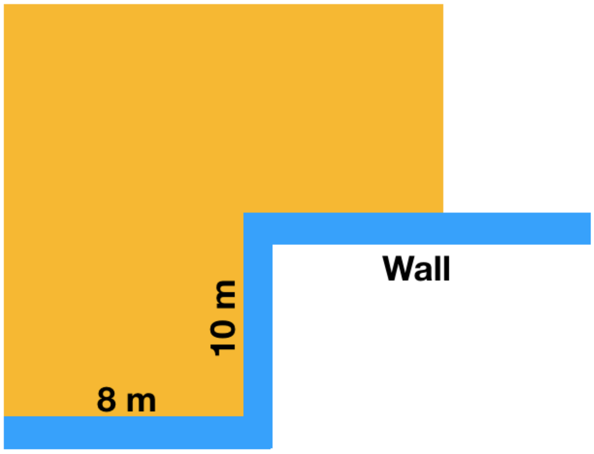
QUESTION TWO

THE FARMER'S FENCE (9 MARKS)

(a) A farmer has 40m of fencing. What is the largest rectangular enclosure he can create using this fencing? (3 marks)

(b) If you could curve the fence, would this increase the area of the enclosure? Provide an explanation. (2 marks)

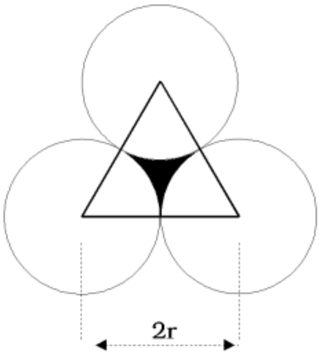
(c) The farmer has a wall as shown below at another part of his property. What is the largest area that could be enclosed using the 40m of fencing and the wall? (4 marks)

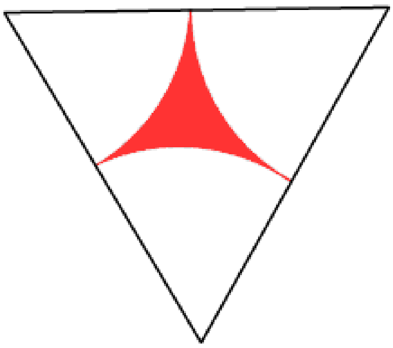


QUESTION THREE

FASHIONABLE FLOORING (6 MARKS)

Carla would like to pave her new outdoor landscape with tiles that are equilateral triangles and contain the pattern show below on each tile. The tile pattern was made by overlaying three circles as shown.



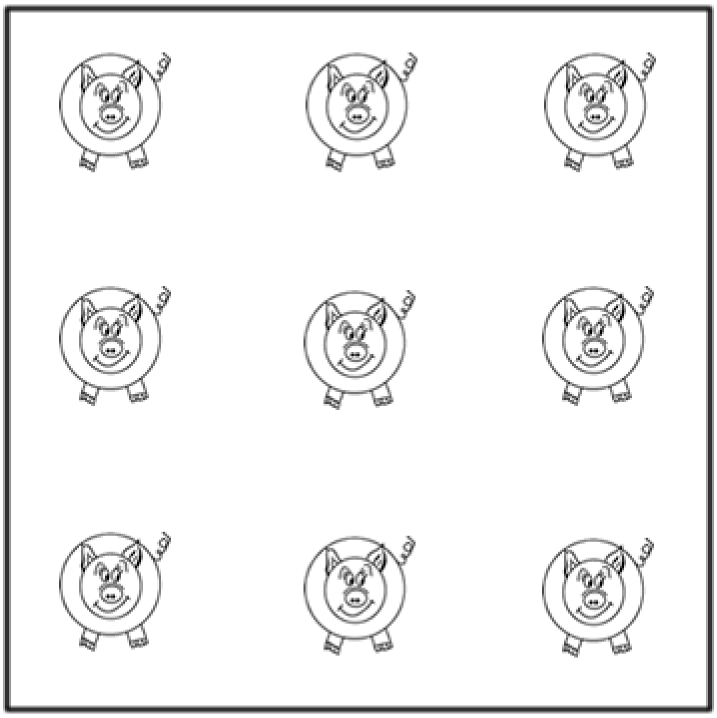


What proportion of the flooring will be shaded?

QUESTION FOUR

PIG PENS (10 MARKS)

(a) 9 pigs are roaming a field. The property owner would like to place two squares of fencing such that each pig is contained within its own area. Sketch two squares on the image below to show how the farmer could do this. (3 marks)



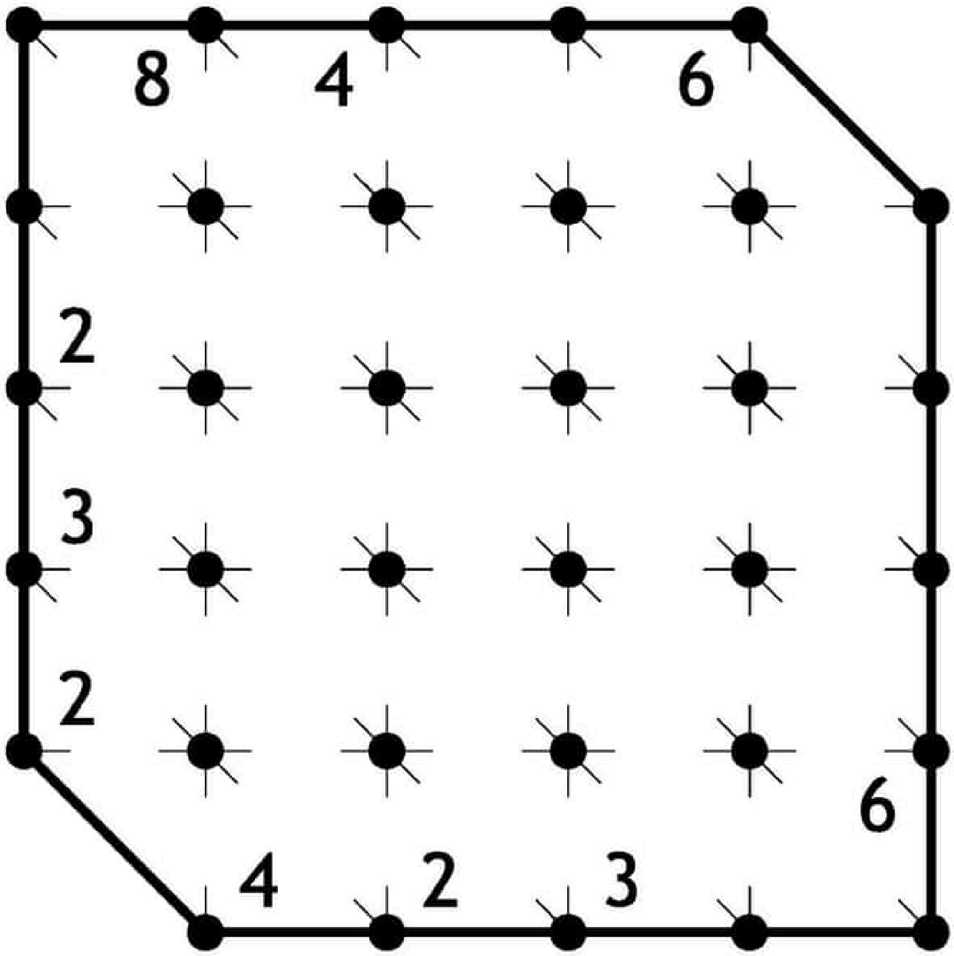
(b) The owner has now tasked you with a more challenging arrangement of fencing. Draw fences (vertically, horizontally or diagonally) between the posts (black dots) below, such that:

i. every post connects exactly three fences;

ii. an enclosure made of a single triangle has no number inside it;

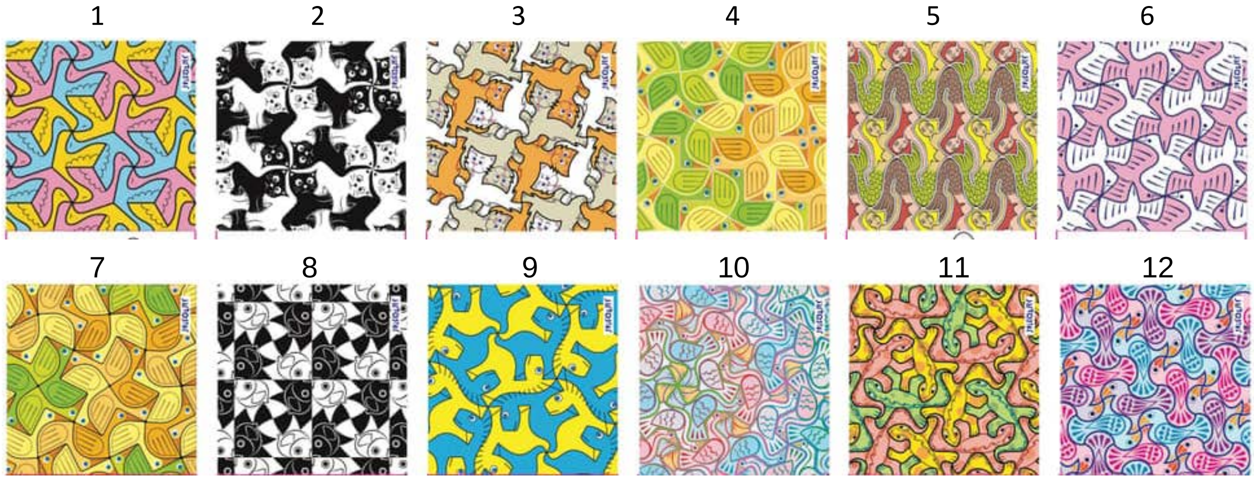
iii. an enclosure made out of more than a single triangle contains the number of triangles inside it; and

iv. no enclosure contains more than one number in it. (7 marks)



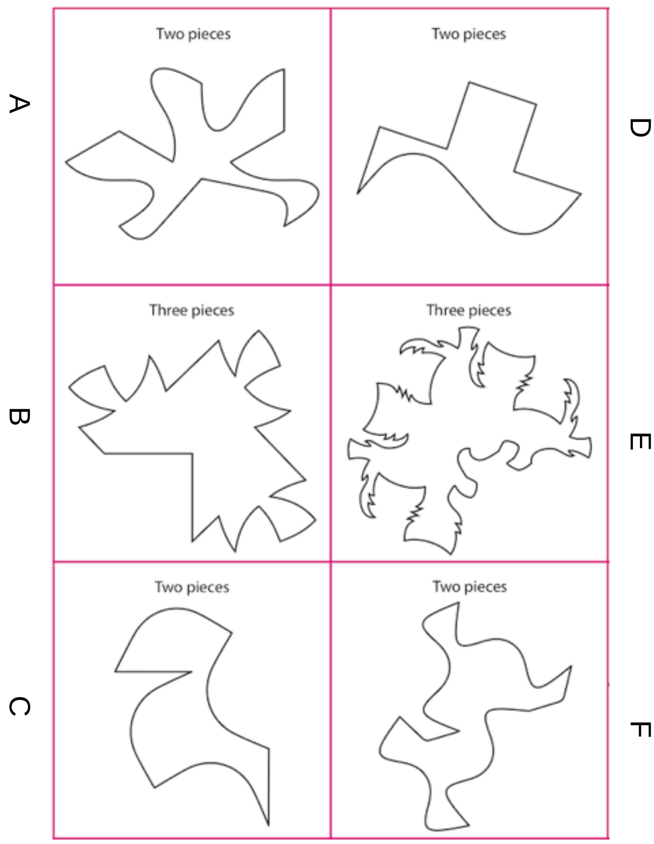
QUESTION FIVE

LUDICROUS LANDSCAPES (15 MARKS)

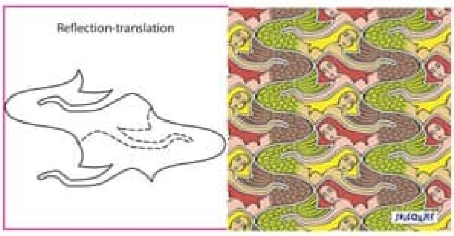


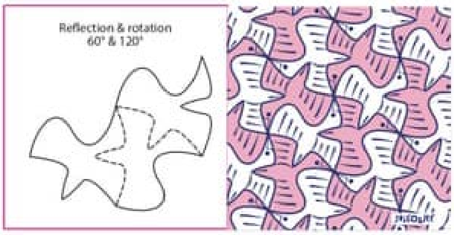
Above are a series of hypnotic landscapes formed by tessellating some of the unusual shapes below. Each shape has been repeated using symmetry rules (e.g. rotation and translation) to form the patterns.

(a) A critical step in being able to undertake symmetry-based operations is first determine the axis of symmetry in a shape. For each shape below, draw either one or two lines to cut the shape into the number of ***equal*** pieces as indicated (6 marks).



(b) For each of the above shapes (A-F), your task is to now assign which pattern above was created using that shape (1-12). Then describe the symmetry operation taken to form the pattern. Two examples are provided below to guide you. The possible operations for your shapes in part (a) are rotation (mostly by common angles such as 90° or 180°) and reflection (9 marks).





PATTERN NUMBER (1-12)	SHAPE (A-F)	SYMMETRY OPERATION

QUESTION SIX

MARCH OF THE ANTS (4 MARKS)

Three ants are each stationary at a separate corner of a triangle. Each ant selects one direction at random and starts walking along the triangle's edge in the chose direction. By considering how many outcomes are possible, what is the probability that none of the ants will collide if they do not change direction again?

QUESTION SEVEN

PREDICTING TREE GROWTH (10 MARKS)

A botanist wants to model the growth of a new tree she planted. When planted, the tree was 86 cm. One year after plantation, the tree grew 42 cm. Each following year the tree grew in height by 95% of the growth in the previous year.

a) What will the tree's height be at the end of the second year? (1 mark)

b) What will tree's height be at the end of the third year and fourth year? (2 marks)

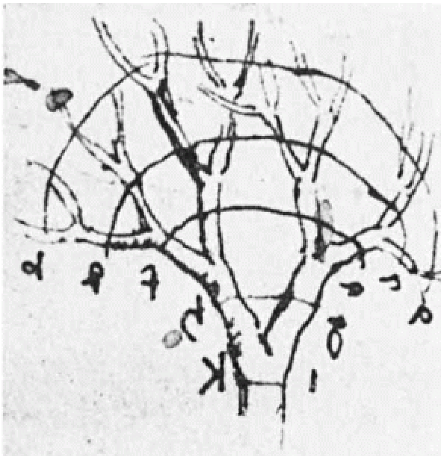
c) Identify the pattern and calculate *a* and *b* if the height at the end of *n* years is to be expressed as Height = 86 + $a + ar + ar^2 + ar^3 + ... + ar^{n-1}$. (2 marks)

d) By multiplying the equation in (c) by *r*, and then subtracting this new equation from the equation in (c), determine a simple expression for the Height of the tree after the *n*th year. (3 marks)

e) As *n* approaches infinity, will the height of the tree also approach infinity, or is there a limit? Include calculations to justify your answer and determine the value of the limit if you believe it exists. (2 marks)

QUESTION EIGHT

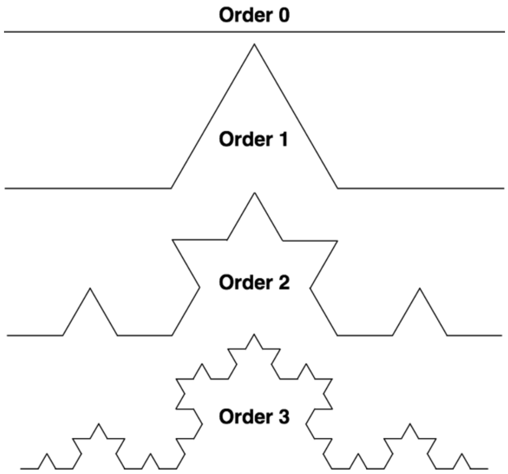
DA VINCI'S FRENETIC FRACTALS (25 MARKS)



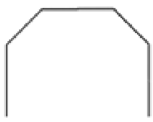
Leonardo da Vinci developed a rule that states when a tree trunk divides, the cross-sectional area of all the branches at that degree of division add up to the cross-sectional area of the trunk. When those branches then divide, the sum of all those areas also equals that of the trunk, and so forth.

(a) You know that the trunk of a tree has a cross section area of 120cm^2. At the second tier of the tree there are 3 branches. One branch has a radius of 2cm, while a second branch has a radius of 3cm. Calculate the radius of the third branch. Note: Area of a circle $= \pi r^2$. (3 marks)

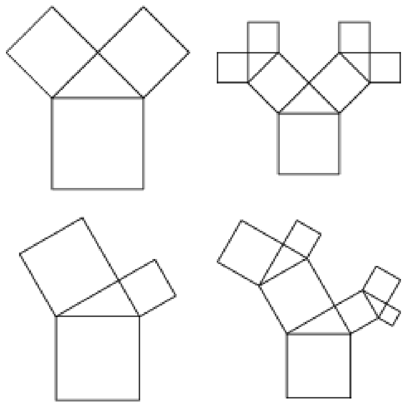
(b) Mathematicians have looked more closely at tree growth and discovered that many leaves grow as a fractal. An example of a fractal pattern is the snowflake, seen below. A fractal is created by repeatedly replacing each segment (line/edge) of a generated shape with a smaller copy of the generator. For example, if you start with a straight line and insert a two-sided triangle, you produce the following fractal after 3 repetitions (order 3):



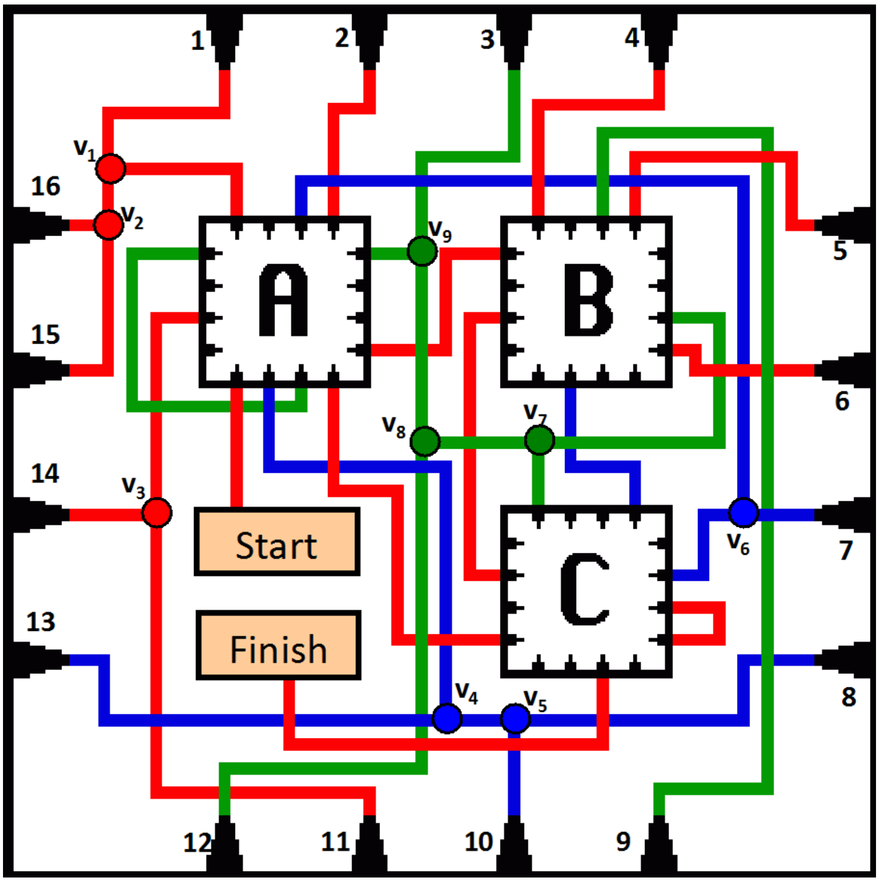
(c) Consider the shape below, an order 1 fractal. Here, the order 0 state was an empty plane, not a straight line. Your task is to produce the fractal of order 2 and 3 by using the shape itself as the generator (i.e. what you iterate on each side of the shape). (3 marks)



(d) So far, the fractals you have seen are symmetrical. Below are the starts of two asymmetrical fractals that are used to model tree branches. By considering the change made between the initial and order 1 fractals, sketch the order 2 fractals for each. (4 marks)



(e) In the 1950s, a mathematician – Mark Wolf – created a fractal maze. His fractal maze is shown below. The idea was to generate a maze such that when you moved inside a smaller box ('A', 'B' or 'C') you enter an entire new copy of the maze. Specifically, you enter the maze at the same numbered point as the smaller point you entered the smaller box on. For example, if you travel along the red edge from the start and enter A, inside A you are then on the green path at 12. From here, you can travel to vertex $\mathbf{V_8}$. You could then move to $\mathbf{V_7}$, enter C, enter B or take a different path entirely! Remember every time you enter a new iteration of the maze you must leave that iteration and the iterations before it to reach the base maze where the finish is.



Your challenge is to find a path from the start to finish (both on the base maze) and complete the 17 moves that are required to do so in the table on page 18. Some moves have been provided for you in the table already. For each move, indicate the start point and end point along with the iteration(s) of the maze you are in. (15 marks)

Hint: Consider the 'prime' paths that you can take, being the paths in the base maze that avoid entering an iteration of the maze (e.g. 12-3). In this question, these are like our prime numbers. These become critical when you try to move from other iterations of the maze.

MOVE	START FROM X	ENTER Y	ITERATION STACK
1	Start	A12	-
2			A
3	A3	C6	
4			
5			
6			
7			CBAA
8			
9	A10		
10		1	
11			
12			C
13	8		
14			CAA
15			
16			
17	C10	Finish	-