



STATE DA VINCI DECATHLON 2017

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS
IN YEARS 5 & 6



MATHEMATICS

TEAM NUMBER _____

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
/6	/16	/5	/10	/8	/5	/6	/56

QUESTION 1: POWER UP

(6 MARKS)

James and Joseph competed in a running race. At the time James completed the race, after 32 minutes, Joseph had only travelled $\frac{4}{9}$ ths of the course. Joseph's average speed for the race was 65 metres per minute less than James' speed.

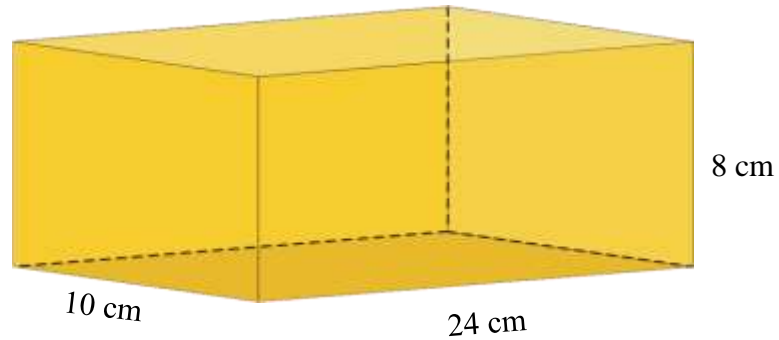
- a) Using $speed = \frac{distance}{time}$, $distance = speed \times time$ or $time = \frac{distance}{speed}$ calculate the distance of the entire course. (4 marks)

- b) Calculate the average speed at which James ran the race in metres/minute (2 marks)

QUESTION 2: CUTTING CUBES

(16 MARKS)

Tara has a block of wood as shown above. She would like to use her powersaw to cut as many $3\text{cm} \times 3\text{cm} \times 3\text{cm}$ cube pieces as possible from the block of wood so she can build a tower.



a) How many 3cm cubes can Tara cut from this wood block? (1 mark)

b) How tall will her tower be using these blocks if she builds one on-top of another? (1 mark)

- c) Tara wants to place the blocks into triangular layers to build the tower: 1 on top, 3 in the next row, 6 in the following, 10 in the fourth row and so on. Layers 1 and 2 have been drawn below, with a dot representing a cube. Sketch layers 4-5 (2 marks)



- d) After drawing the above shapes Tara assumes there will be 6 layers in her tower and starts building the tower from the bottom row. Will she have enough cubes? (2 marks)

- e) Imagine in each layer that the current triangle you have drawn is reflected along the diagonal line on the right of the triangle, as shown below for layer 2. Draw similar triangles for layers 3-4 below. (2 marks)



- f) Using the new pattern above, write an equation using n for how many cubes are in the n th layer. (3 marks)

- g) Tara would like to build a tower using the triangular building method in part (c). The tower is to be 42cm high.
- (i) Determine the number of cubes Tara requires to build the tower. (2 marks)
- (ii) Only large cubic blocks of wood are available for Tara to cut to form the smaller cubes. Calculate the dimensions of the smallest possible original wood piece that would allow Tara to build the 42cm tower using this building design. (3 marks)

QUESTION 3: POWER LOSS

(5 MARKS)

A particular generator can produce 100 KW of power per hour.

(5 marks)

- Once enough power is produced, the power is sent to a storage facility through a cable that loses 12% of the power.
- At the storage facility, 12 KW of power are lost to the environment every 30 minutes.
- The power is then transported from the storage facility to a local factory, with only 58% of the power sent being received by the factory.

The generator produces 12000 KW of power. It is known the storage facility held the power from the generator for 2 days.

Calculate the amount of power that will reach the factory.

QUESTION 4: POWERFUL PLANET

(10 MARKS)

The power released by a star can expressed by,

$$P = A\varepsilon\sigma T^4$$

where P is power in Watts, A is the surface area of the sun in square metres, T is the temperature (in Kelvin), $\sigma = 5.670373 \times 10^{-8} \text{ W/m}^2\text{K}^4$ and $\varepsilon = 1$.

- (a) The surface area of a sphere, A, is given by $A = 4\pi r^2$. (2 marks)

A star called AXP-3310 has a radius (r) of 780,300 km. Calculate the surface area of AXP-3310.

- (b) The temperature of the star called AXP-3310 is 6280 Kelvin. (2 marks)

Calculate the power that is released by AXP-3310 and provide your answer in the form $a.bcd \times 10^x$.

- (c) The temperature of AXP-4000 is 4000 K and the power it produces is $7.10 \times 10^{18} \text{ W}$. (3 marks)

Find the surface area.

- (d) The radius of star AXP-3300 is Y . (3 marks)
The radius of star AXP-3301 is $2Y$.

How much more power will be released by AXP-3301 than AXP-3300?
Write your answer in the form, for example, '12x more than AXP-3300'.

QUESTION 5: POWER NAP

(8 MARKS)

- (a) (4 marks)
- Every 1 in 2 days Louise is out of bed at 8am.
 - Every 1 in 4 days Louise is out of bed at 9am.
 - Every 1 in 4 days Louise is out of bed by 10 am.

In a period of 7 days, how many days will Louise **definitely** be out of bed by 9:30 am?

- (b) (4 marks)
- Joe starts his morning with an energy bar. He has three protein bars to choose from:
- Bar A: costs \$3.20 and provides 870 J of energy.
Bar B: costs \$2.80 and provides 800 J of energy.
Bar C: costs \$4.80 and provides 1300 J of energy.

Which energy bar provides the best energy value for money? Show calculations to support your conclusion.

QUESTION 6: POWER RULES**(5 MARKS)**

The following rules are known in Mathematics:

Assume $x \neq 0$

$$x^a x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

$$x^0 = 1$$

$$\frac{1}{x} = x^{-1}$$

$$\frac{1}{x^a} = x^{-a}$$

Use the rules to simplify the following:

(a) $x^3 x^7$ (1 mark)

(b) $\frac{x^6}{x^3}$ (1 mark)

(c) $x^y (x^y)^{zx}$ (1 mark)

(d) $\frac{xx^2}{x^3}$ (2 marks)

QUESTION 7: A PIE OF POWER

(6 MARKS)

The following table illustrates the percentage of energy produced from different sources.

First filling in the missing data, create a pie chart that visually illustrates this data. Hint: You will need to use a protractor and remember that a circle has 360 degrees (=100%). (5 marks)

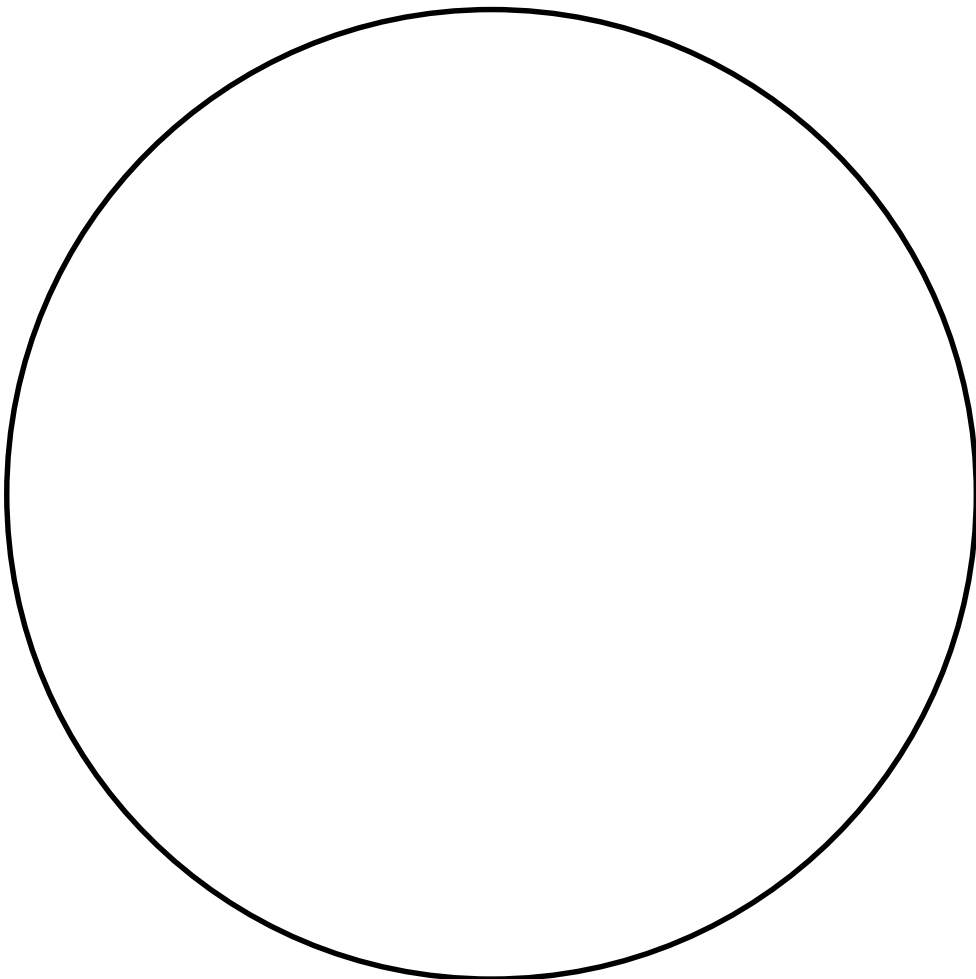
- (a) Fill in the missing data show working in the space below the table
(Remove the answers on Question sheet)

(3 marks)

Source	% produced
Wind	7
Solar	
Hydro	23
Coal	42
Geothermal	8
Gas (three times as much as solar power)	

- (b) Create a pie chart that visually illustrates this data, by completing the table below. (3 marks)
 Hint: You will need to use a protractor and remember that a circle has 360 degrees (=100%).

Source	% produced	Angle
Wind	7	
Solar	5	
Hydro	23	
Coal	42	
Geothermal	8	
Gas (three times as much as solar power)	15	





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MATHEMATICS SOLUTIONS

TEAM NUMBER _____

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
/6	/16	/5	/10	/8	/5	/6	/56

QUESTION 1: POWER UP**(6 MARKS)**

James and Joseph competed in a running race. At the time James completed the race, after 32 minutes, Joseph had only travelled $\frac{4}{9}$ ths of the course. Joseph's average speed for the race was 65 metres per minute less than James'.

- a) Using $speed = \frac{distance}{time}$, $distance = speed \times time$ or $time = \frac{distance}{speed}$ (4 marks)
calculate the distance of the entire course.

Solution	Marks	
<p>The space between James and Joseph will increase 65 m/min. After 32 minutes, the space between both will be $65 \times 32 = 2080$ m At this time, Joseph had travelled $\frac{4}{9}$ths of the course which means 2080 must be $\frac{5}{9}$ths of the course. $\frac{2080}{5} = 416$ Therefore, the distance of the entire course must be $416 \times 9 = 3744$</p>	Award 4	For correct answer
	Award 3	One error
	Award 2	Two errors
	Award 1	Three errors

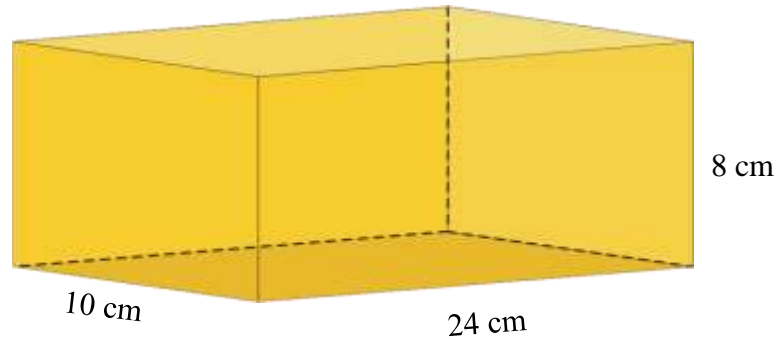
- b) Calculate the average speed at which James ran the race in metres/minute (2 marks)

Solution	Marks	
<p>$s = \frac{2079.999}{32}$ $s = 65 \text{ metres/minute}$ $S = d/t = 2079.999/32 \text{ minutes} = 65 \text{ m/min}$</p>	Award 2	For correct answer
	Award 1	One error

QUESTION 2: CUTTING CUBES

(16 MARKS)

Tara has a block of wood as shown above. She would like to use her power saw to cut as many $3\text{cm} \times 3\text{cm} \times 3\text{cm}$ cube pieces as possible from the block of wood so she can build a tower.



- a) How many 3cm cubes can Tara cut from this wood block? (1 mark)

Solution	Marks	
$3 \times 8 \times 2 = 48$	Award 1	For correct answer

- b) How tall will her tower be using these blocks if she builds one on-top of another? (1 mark)

Solution	Marks	
$48 \times 3 = 144 \text{ cm}$	Award 1	For correct answer

- c) Tara wants to place the blocks into triangular layers to build the tower: 1 on top, 3 in the next row, 6 in the following, 10 in the fourth row and so on. Layers 1 and 2 have been drawn below, with a dot representing a cube. Sketch layers 4-5 (2 marks)



Solution	Marks	
	Award 2	Both layers 4 and 5 correct
	Award 1	One layer only correct or shown

- d) After drawing the above shapes Tara assumes there will be 6 layers in her tower and starts building the tower from the bottom row. Will she have enough cubes? (2 marks)

Solution	Marks	
<p>$1 + 3 + 6 + 10 + 15 + 21$ \therefore He will need 56 cubes, he does not have enough he will be 8 cubes short.</p>	Award 2	For correct answer including conclusion and evidence of correct calculation
	Award 1	For correct calculation with no conclusion or incorrect conclusion.
	Award 0	

- e) Imagine in each layer that the current triangle you have drawn is reflected along the diagonal line on the right of the triangle, as shown below for layer 2. Draw similar triangles for layers 3-4 below. (2 marks)



Solution	Marks	
	Award 2	Both layers 3 and 4 correct
	Award 1	One layer only correct or shown

- f) Using the new pattern above, write an equation using n for how many cubes are in the n th layer. (3 marks)

Solution	Marks	
<p>Cubes in each layer $= n \times (n - 1)$ \therefore Cubes in n^{th} layer $= \frac{n \times (n - 1)}{2}$</p>	Award 3	For correct answer equation.
	Award 2	For correct with one error ie finding number in a rectangle
	Award 1	For evidence of the concept of halving only.

- g) Tara would like to build a tower using the triangular building method in part (c). The tower is to be 42cm high.
 (i) Determine the number of cubes Tara requires to build the tower. (2 marks)

Solution	Marks																													
<p>Height of 42 = 14 layers.</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr> <tr><td>1</td><td>3</td><td>6</td><td>10</td><td>15</td><td>21</td><td>28</td><td>36</td><td>45</td><td>55</td><td>66</td><td>78</td><td>91</td><td>105</td></tr> </table> <p>Total blocks =560</p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	3	6	10	15	21	28	36	45	55	66	78	91	105	Award 2	For correct answer
	1	2	3	4	5	6	7	8	9	10	11	12	13	14																
1	3	6	10	15	21	28	36	45	55	66	78	91	105																	
	Award 1	For finding the number of layers only																												

(ii) Only large cubic blocks of wood are available for Tara to cut to form the smaller cubes. Calculate the dimensions of the smallest possible original wood piece that would allow Tara to build the 42cm tower using this building design. (3 marks)

Solution	Marks	
<p>560 blocks required</p> <p>Cubic block possibilities: $8 \times 8 \times 8 = 512$ (<i>too small</i>) $9 \times 9 \times 9 = 729$ (<i>works</i>) \therefore A block $9 \times 9 \times 9$ is required.</p>	Award 3	For correct answer
	Award 2	For finding the total number of blocks only
	Award 1	Completing the table only

QUESTION 3: POWER LOSS**(5 MARKS)**

A particular generator can produce 100 KW of power per hour.

(5 marks)

- Once enough power is produced, the power is sent to a storage facility through a cable that loses 12% of the power.
- At the storage facility, 12 KW of power are lost to the environment every 30 minutes.
- The power is then transported from the storage facility to a local factory, with only 58% of the power sent being received by the factory.

The generator produces 12000 KW of power. It is known the storage facility held the power from the generator for 2 days.

Calculate the amount of power that will reach the factory.

Solution	Marks	
<i>Power from station = 88% × 12000 = 10560</i>	Award 5	For correct answer
<i>Power lost at storage = 96 × 12 = 1152 lost</i>	Award 4	For one error only
<i>1056 – 1152 = 9408</i>	Award 3	For two errors or omissions
<i>58% × 9408 = 5456.64kW</i>	Award 2	For three errors or omissions
	Award 1	For four errors or omissions

QUESTION 4: POWERFUL PLANET**(10 MARKS)**

The power released by a star can expressed by,

$$P = A\epsilon\sigma T^4$$

where P is power in Watts, A is the surface area of the sun in square metres,
T is the temperature (in Kelvin), $\sigma = 5.670373 \times 10^{-8} \text{ W/m}^2\text{K}^4$ and $\epsilon = 1$.

- (a) The surface area of a sphere, A, is given by $A = 4\pi r^2$. (2 marks)

A star called AXP-3310 has a radius (r) of 780,300 km. Calculate the surface area of AXP-3310.

Solution	Marks	
$A = 4 \times \pi \times 780300^2$ $A = 7.6512621 \times 10^{12} \text{ square metres}$	Award 2	For correct answer.
	Award 1	For evidence of correct substitution only.

- (b) The temperature of the star called AXP-3310 is 6280 Kelvin. (2 marks)

Calculate the power that is released by AXP-3310 and provide your answer in the form a.bcd x 10^x.

Solution	Marks	
$P = 7.65 \times 10^{12} \times 5.670373 \times 10^{-8} \times 6280^4$ $P = 6.747 \times 10^{20}$	Award 2	For correct answer must be in correct notation.
	Award 1	For evidence of correct substitution with incorrect answer or incorrect notation.

- (c) The temperature of AXP-4000 is 4000 K and the power it produces is 7.10×10^{18} W. (3 marks)

Find the surface area.

Solution	Marks	
$7.10 \times 10^{18} = A \times 5.670373 \times 10^{-8} \times 4000^4$ $7.10 \times 10^{18} = A \times 14516154.88$ $4.891102402 \times 10^{11} = A$ $489110240191 = A$	Award 3	For correct answer must be in correct notation.
	Award 2	For evidence of correct substitution with one subsequent error.
	Award 1	For evidence of correct substitution only.

- (d) The radius of star AXP-3300 is Y. (3 marks)
The radius of star AXP-3301 is 2Y.

How much more power will be released by AXP-3301 than AXP-3300?
Write your answer in the form, for example, '12x more than AXP-3300'.

Solution	Marks													
<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;">Star AXP-3300</td> <td style="width: 50%; border: none;">Star AXP-3300</td> </tr> <tr> <td style="border: none;">$SA = 4 \times Y^2$</td> <td style="border: none;">$SA = 4 \times (2Y)^2$</td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">$SA = 4 \times 4Y^2$</td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">$SA = 16Y^2$</td> </tr> <tr> <td style="border: none;">$Effect\ size = \frac{16y^2}{4y}$</td> <td style="border: none;"></td> </tr> <tr> <td style="border: none;">$Effect\ size\ 4X$</td> <td style="border: none;"></td> </tr> </table>	Star AXP-3300	Star AXP-3300	$SA = 4 \times Y^2$	$SA = 4 \times (2Y)^2$		$SA = 4 \times 4Y^2$		$SA = 16Y^2$	$Effect\ size = \frac{16y^2}{4y}$		$Effect\ size\ 4X$		Award 3	For correct answer.
	Star AXP-3300	Star AXP-3300												
	$SA = 4 \times Y^2$	$SA = 4 \times (2Y)^2$												
	$SA = 4 \times 4Y^2$													
	$SA = 16Y^2$													
$Effect\ size = \frac{16y^2}{4y}$														
$Effect\ size\ 4X$														
	Award 2	For two expressions of power with no conclusion.												
	Award 1	For one expression of power.												

QUESTION 5: POWER NAP**(8 MARKS)**

- (a) (4 marks)
- Every 1 in 2 days Louise is out of bed at 8am.
 - Every 1 in 4 days Louise is out of bed at 9am.
 - Every 1 in 4 days Louise is out of bed by 10 am.

In a period of 7 days, how many days will Louise **definitely** be out of bed by 9:30 am?

Solution	Marks	
$\frac{1}{2} \times 7 = 3.5 \text{ days}$ $\frac{1}{4} \times 7 = 1.75 \text{ days}$ Total = 5.25 days We can't be sure of the .25 so there are 5 days definitely she will be out of bed by 9:30am	Award 4	For correct answer must be correctly rounded.
	Award 3	One error in calculation or conclusion.
	Award 2	Two errors in calculation or conclusion.
	Award 1	Three errors in calculation or conclusion.

- (b) Joe starts his morning with an energy bar. He has three protein bars to choose from: (4 marks)
- Bar A: costs \$3.20 and provides 870 J of energy.
 Bar B: costs \$2.80 and provides 800 J of energy.
 Bar C: costs \$4.80 and provides 1300 J of energy.

Which energy bar provides the best energy value for money? Show calculations to support your conclusion.

Solution	Marks	
$870 \div 3.20 = 271.88 \text{ J}/\$$ $800 \div 2.80 = 285.71 \text{ J}/\$$ $1300 \div 4.80 = 2710.83 \text{ J}/\$$ $\therefore \text{Bar B provides the most energy per } \$$	Award 4	For a correct conclusion from correct calculations.
	Award 3	One error in calculation or conclusion.
	Award 2	Two errors in calculation or conclusion.
	Award 1	Three errors in calculation or conclusion.

QUESTION 6: POWER RULES**(5 MARKS)**

The following rules are known in Mathematics:

Assume $x \neq 0$

$$x^a x^b = x^{a+b}$$

$$x^0 = 1$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{1}{x} = x^{-1}$$

$$(x^a)^b = x^{ab}$$

$$\frac{1}{x^a} = x^{-a}$$

Use the rules to simplify the following:

(a) $x^3 x^7$

(1 mark)

Solution	Marks	
x^{10}	Award 1	For correct answer

(b) $\frac{x^6}{x^3}$

(1 mark)

Solution	Marks	
x^3	Award 1	For correct answer

(c) $x^y (x^y)^{zx}$

(1 mark)

Solution	Marks	
x^{y+yzx}	Award 1	For correct answer or equivalent expression

(d) $\frac{xx^2}{x^3}$

(2 marks)

Solution	Marks	
$\frac{x^3}{x^3} = 1$	Award 2	For correct answer
	Award 1	For answer not fully simplified

QUESTION 7: A PIE OF POWER**(6 MARKS)**

The following table illustrates the percentage of energy produced from different sources.

First filling in the missing data, create a pie chart that visually illustrates this data. Hint: You will need to use a protractor and remember that a circle has 360 degrees (=100%). (5 marks)

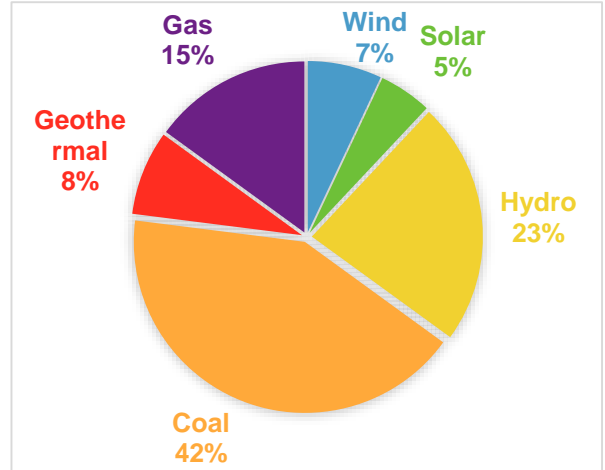
- (a) Fill in the missing data show working in the space below the table (3 marks)
(Remove the answers on Q sheet)

Source	% produced
Wind	7
Solar	5
Hydro	23
Coal	42
Geothermal	8
Gas (three times as much as solar power)	15

Solution	Marks	
$7+23+42+8=80\%$ $20\% \div 4 = 5$ $\therefore \text{Solar} = 5\%$ $\therefore \text{Gas} = 3 \times 5 = 15\%$	Award 3	For both answers correct.
	Award 2	For one error or omission.
	Award 1	For two errors or omissions.

- (b) Create a pie chart that visually illustrates this data, by completing the table below. (3 marks)
 Hint: You will need to use a protractor and remember that a circle has 360 degrees (=100%).

Source	% produced	Angle
Wind	7	$7\% \times 360 = 25.2\%$
Solar	5	$5\% \times 360 = 18\%$
Hydro	23	$23\% \times 360 = 82.8\%$
Coal	42	$42\% \times 360 = 151.2\%$
Geothermal	8	$7\% \times 360 = 28.8\%$
Gas (three times as much as solar power)	15	$15\% \times 360 = 54\%$



Solution	Marks	
<i>See table above</i>	Award 3	Correct angle calculations and correct chart.
	Award 2	For one error
	Award 1	For two errors

